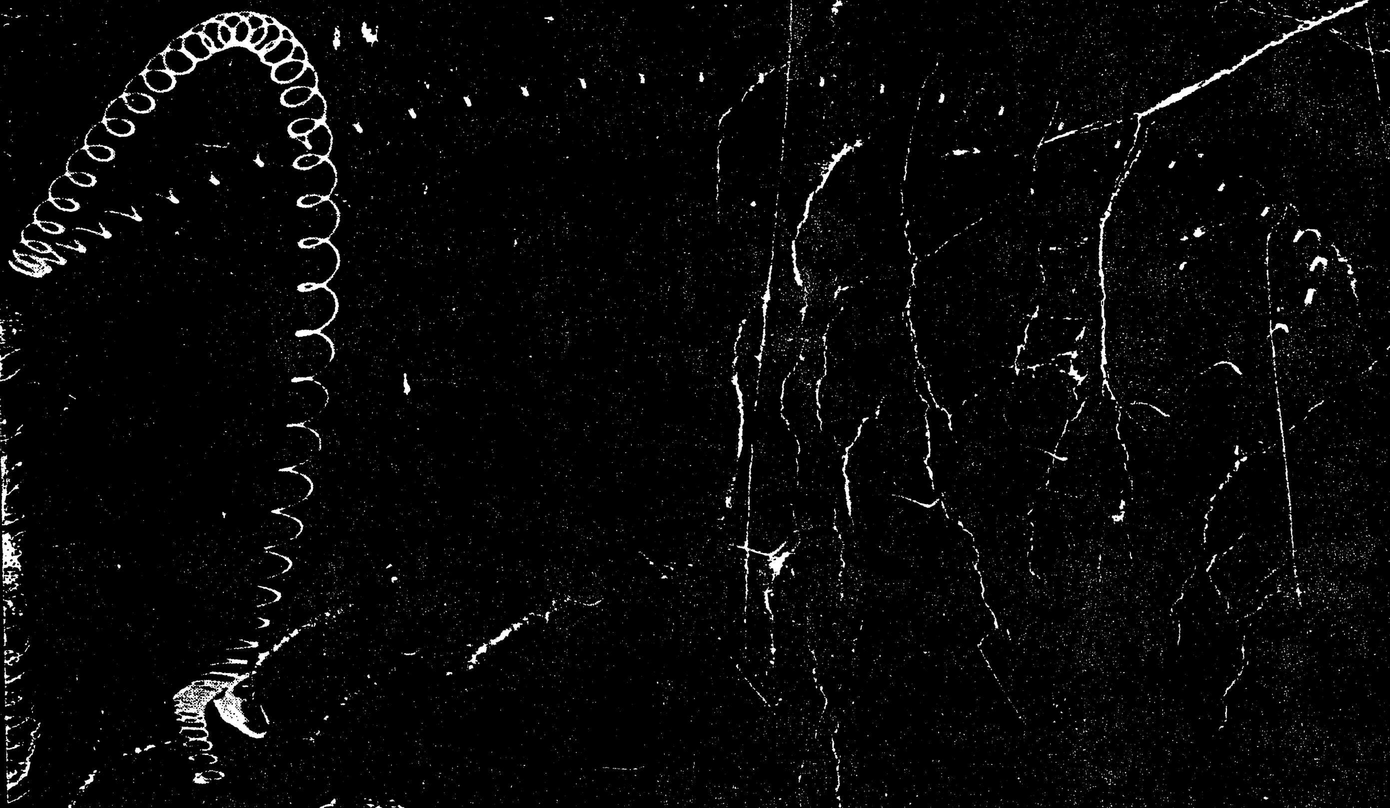
BOOMERANGS, AERODYNAMICS AND MOTION



FELIX HESS

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BOOMERANGS, AERODYNAMICS AND MOTION

RIJKSUNIVERSITEIT TE GRONINGEN

BOOMERANGS, AERODYNAMICS AND MOTION

PROEFSCHRIFT

TER VERKRIJGING VAN HET DOCTORAAT
IN DE WISKUNDE EN NATUURWETENSCHAPPEN
AAN DE RIJKSUNIVERSITEIT TE GRONINGEN
OP GEZAG VAN DE RECTOR MAGNIFICUS DR. A. WATTEL
IN HET OPENBAAR TE VERDEDIGEN
OP MAANDAG 7 JULI 1975
DES NAMIDDAGS TE 2.45 UUR PRECIES

DOOR

FELIX HESS
geboren te 's-Gravenhage

Druk: Verenigde Reproduktie Bedrijven – Kleine der A 4 – Groningen

Promotor: Prof.Dr. J.A. Sparenberg
Coreferent: Prof.Dr. H. de Waard

Stellingen behorende bij het proefschrift van Felix Hess: Boomerangs, aerodynamics and motion.

1. Wetenschappelijk onderzoek aan de aerodynamica van helicopter rotors vormt een duidelijke bijdrage tot de technologische wapenwedloop.

2. Bij de in de dragende-vlak theorie gebruikelijke collocatie methode mag men niet verwachten dat, bij toenemend aantal termen in de reeksontwikkeling van de lift functie en toenemend aantal collocatie punten in koorde richting, de oplossing convergeert naar een lift verdeling die voldoet aan de Kutta voorwaarde.

3. Als het in dit proefschrift ontwikkelde boemerang model wordt vereenvoudigd door de geïnduceerde snelheid van de lucht te verwaarlozen, kunnen er toch nog redelijke boemerang banen mee worden berekend.

4. Beschouw een lichaam dat bestaat uit een rechte, dunne, homogene staaf die in het midden geknikt is over een hoek α ($0 \leq \alpha \leq \pi$). Zij m de massa en I het grootste hoofdtraagheidsmoment van dit lichaam, en a de afstand van de uiteinden van het lichaam tot het massa middelpunt. Dan is $I = \frac{1}{3} ma^2$.

5. De waarneming en beschrijving van boemerangs door volkenkundigen is soms op onjuiste wijze beïnvloed door informatie afkomstig van natuurkundigen.

6. Er zijn ethnocentrische trekken aanwezig in een groot deel van de bestaande literatuur over de oorsprong van boemerangs.

7. De kans op een windstille zomernacht in noord-oost Nederland is groter na een dag met westelijke wind dan na een dag met oostelijke wind.

8. Men kan twee stipjes op een treinruit aanbrengen zodanig dat men ze waarneemt als één stip die in het landschap met de trein meebeweegt op een afstand S van bijv. 100 à 200 meter. Bij het passeren van objecten zoals huizen en bomengroepen op een afstand aanmerkelijk kleiner dan S, ziet men de stip plotseling naderbij komen, rakelings vóór het passerende object langs gaan en weer terugkeren tot zijn oude positie. (Deze observatie stemt overeen met de opmerking van Ogle: "... it is to be expected that in those surroundings that have been artificially produced to provide a conflict between stereoscopic stimuli and empirical factors, the meaningless stimuli may be suppressed by the meaningful, that is, by the perceptions from the empirical motives for depth." [K.N. Ogle: "Theory of stereoscopic vision." in: S. Koch, ed. "Psychology: a study of science." (McGraw-Hill, New York, 1959) p. 362-394].)

9. De beeldscherpte van een fototoestel met open diafragma, dat is ingesteld volgens de aangegeven afstandsschaal, kan worden verbeterd door de afstandsinstelling te wijzigen.

10. De verslaggeving in kranten van verkeersongevallen waarbij een rijdende auto en een lopend kind betrokken zijn is doorgaans niet alleen eenzijdig, maar draagt bovendien bij tot de bestendiging van de bestaande verkeersonveiligheid.

11. Bij het totstandkomen van de schilderijen van Han Jansen speelt sinds 1973 de zwaartekracht een essentiële rol.

12. Een half-bolvormige wollen muts kan spiraalsgewijs worden gehaakt als volgt. Stel R = de straal van de halve bol, h = de hoogte van de haaksteek, t = het aantal toeren, gerekend vanaf de kruin. Begin bij de kruin, en meerder telkens één steek op de n(t) steken, waarbij

$$n(t) \approx \frac{R}{h} \operatorname{tg} \frac{ht}{R}$$

Ga zo door totdat $t \approx \frac{\pi R}{2h}$ en hecht af.

13. De veel voorkomende mening dat leervakken of wetenschappelijke disciplines "moeilijker" zijn naarmate ze exacter zijn berust vaak op een onderschatting van de complexiteit van de problemen waarmee minder exacte vakken zich bezighouden.

14. Het geschatte aantal door intelligente wezens bewoonde planeten buiten ons zonnestelsel zal vooralsnog blijven toenemen.

15. Dit is een stelling.

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Théorèmes de la thèse de Felix Hess: Boomerangs, aerodynamics and motion.

1) La recherche concernant l'aerodynamique des rotors d'hélicopter contribue largement à la course aux armements.

2) Dans le cas où on augmente les termes d'une progression de la fonction de montée et les points de collocation dans la direction de corde, la méthode de collocation, utilisée habituellement dans la théorie de la surface portante, n'a pas comme conséquence une distribution de montée qui remplisse la condition de Kutta.

3) On peut simplifier le modèle de boomerang développé dans cette thèse en supposant que la vitesse d'introduction de l'air soit négligeable. Les trajectoires de boomerang calculés ainsi sont encore assez correctes.

4) $I = {}^{1}/_{3}\ ma^{2}$ pour un corps qui est constitué d'une barre droite, mince et homogène, fléchie au milieu (angle entre 0 et π). m est la masse, I le moment principal de lenteur du corps et a la distance entre les extrêmes du corps et le milieu de la masse.

5) L'observation et la description de boomerangs par les anthropologues a parfois été influencée d'une façon incorrecte par l'information de la part de physiciens.

6) Une grande partie de la littérature traitant l'origine des boomerangs comporte des éléments ethnocentriques.

7) Les chances d'une nuit d'été sans vent dans le nord-est des Pays-Bas sont plus grandes après une journée de vent d'ouest qu'après une journée de vent d'est.

8) Deux points peuvent être dessinés sur un vitre de train de tel façon que l'oeil les perçoit comme un point qui avance avec le train à une distance S de 100 - 200 mètres. En passant des objets (maisons, groupements d'arbres etc.) qui se trouvent à une distance nettement inférieure à S, on voit le point s'approcher soudainement, frôlant l'objet pour retourner ensuite à l'ancienne position. Cette observation concorde avec la remarque de M. Ogle: ".... it is to be expected that in those surroundings that have been artificially produced to provide a conflict between stereoscopic stimuli and empirical factors, the meaningless stimuli may be suppressed by the meaningful, that is, by the perceptions from the empirical motives for depth." (K.N. Ogle: Theory of stereoscopic vision. in S. Koch, ed "Psychology: a study of science." (McGraw-Hill, New York, 1959) p. 362-394).

9) La netteté de l'image d'un appareil photo à diaphragme ouvert, mise au point selon l'échelle indiquée sur l'appareil, peut être amélioré en changeant la distance.

10) Les reportages dans la presse sur les accidents de la route impliquant une voiture roulante et un enfant à pied sont généralement pas seulement partials mais contribuent en plus à la confirmation du sentiment d'insécurité routière existant.

11) Depuis 1973, la gravitation joue un rôle essentiel à la réalisation des peintures de Han Jansen.

12) Un bonnet mi-sphère peut être crocheté en spiral comme suit. Prenons R = le rayon du demi sphère, h = la hauteur de la maille, t = le nombre de tours comptés du sommet de la tête. Commencez au sommet de la tête et ajoutez un crochet tous les n(t) crochets quand:

$$n(t) = \text{approx. } {}^{R}/_{h} * tg\,({}^{ht}/_{R})$$

Continuez jusqu'a $t = \text{approx. } {}^{\pi R}/_{2h}$ et fixez.

13) L'opinion courante qui consiste à penser que les disciplines (scientifiques) sont plus "difficiles" si elles sont plus exactes est souvent fondée sur la sousestimation de la complexité des problèmes dont s'occupent les disciplines moins exactes.

14) Les estimations du nombre de planètes hors de notre système solaire habitées par des êtres intelligents continuera d'augmenter pour le moment.

15) Ceci est un théorème.

to the Aboriginal people of Australia,
who laid the foundation for this study.

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The narrative may interest you. I found an open space behind a pile of lumber. I looked all about. I was alone. I determined to make a light throw at first, and the first thing that astonished me was the comparatively enormous distance that it travelled upon a slight impulse. Its weight was less than two ounces, and yet it went over 200 feet away. It rose swiftly in the air, whirling and flashing in the sunlight, and, as I thought, extremely beautiful in the graceful ease of its motions. And could I believe my eyes! Yes! It was coming back. It fell within a yard or two of my feet. I picked it up, fully as delighted as ever that black savage could have been who stumbled upon its first discovery, and became a blessing to his race.

[Emerson, 1893, p. 87]

PREFACE

§1. *A study of boomerangs.*

The evolutions which a boomerang is capable of are bewildering, and at first it seems that there could be no possible solution for the extraordinary gyrations it makes; yet in time these become quite clear. [Pern, 1928, p. 102].

Studying the mechanics of returning boomerangs is not different from most other scientific work. But whereas many scientists cannot communicate about the subject of their intense interest with more than a few fellow specialists, the student of boomerangs is in a rather different position, and fortunately so. It is not often nowadays that a subject of scientific research appeals to so many people outside science: anyone who has heard about boomerangs or has seen a boomerang fly and return may feel exactly that curiosity and need for explanation which are at the heart of most scientific work.

Admittedly, this research on boomerangs is probably not characteristic of modern science. The physical principles underlying the boomerang's behaviour have been known for several generations, and part of my work could have been done by others many years ago. For those who in the past turned their attention to the boomerang problem it was usually a pastime, and in the fifty years preceding 1968 hardly any scientific attention at all was paid to boomerangs. Obviously boomerangs are found in an out-of-the-way corner of science rather than in one of its main corridors: the increase of knowledge on the flight of boomerangs may leave the rest of science unchanged. Yet the subject of boomerangs is not quite isolated from all other fields of science; connections with "more serious" topics are unmistakable. The aerodynamics of helicopter rotors is an obvious example. Some of the methods used in this work are similar to those employed elsewhere in aero- and hydrodynamics. It is open to discussion whether this should be considered as an argument in favour of the usefulness of my work. Often the applications of this branch of science fit in a military context, and too often it is taken for granted that such fields of research must be further developed.

The boomerang puzzle is not deep, but it is complex. One must first formalize and reduce it to a problem in applied mathematics, and then find a solution which can be interpreted in terms of physical reality. The complementary part of the boomerang problem involves the design and execution of experiments. But at the very start the main point is to decide what is essential and what can be disregarded.

About ten years ago in the corridors of the physics lab in Groningen, talking with Herman Coster, I suddenly saw how boomerangs can return. (Some people had seen this before, which I did not know). I worked out an extremely simple mathematical model, for which Lanbrecht Kok wrote an Algol program. Rather excited, we plotted by hand the first computed boomerang flight path: it returned! Soon I saw that my equations of motion were those of a spherical pendulum. I tried to improve the model, working on it for one week every two months or so. At that time I still made and threw boomerangs just for fun. Working on a boomerang theory was a pastime too, and so were the first experiments with illuminated boomerangs in 1967. This stage of the boomerang work definitively ended with the publication of the Scientific American article in 1968. By then I had finished my physics study and had started working with Sparenberg at the Department of Applied Mathematics. Boomerangs had become an official subject of study.

First a mathematical model for the aerodynamic forces on boomerangs had to be developed. The basic idea was conceived quickly, but it took three and a half years to work it out and make it produce numerical results. This was highly technical work. The experiments were more diversified as regards the level of sophistication. For instance, there were three weeks of hard laboratory work in Delft with a brand-new wind tunnel and a lot of electronic equipment. In contrast, the boomerang-throwing experiments near Steenwijk were carried out in a pastoral setting. Here a piece of grassland surrounded by trees played the role of a laboratory, and the equipment could be transported by bicycle: homemade boomerangs and wind meters, two cameras, an aluminium ladder, batteries, tiny light bulbs, a 50m tape measure, two fishing rods, pieces of iron wire, etc.

In the evening twilight of a summer night Herman C and I can be seen pushing a perambulator loaded with the above equipment to a place underneath the branches of an oak tree: the origin of our coordinate system. The wind meters are carefully put on top of the fishing rods. A very light breeze makes them spin against the evening sky. (If only the wind would vanish!) Ten field lights are put in their proper positions just above the grass, reminding one of a miniature airfield. Herman installs the cameras while I check the boomerangs. In the meantime it has become dark enough for the experiments to start. Soon a boomerang flies through the air, a whirling trace of light. At fifty metres distance Herman operates the cameras, jots down the wind conditions and shouts directions to prevent me from throwing the boomerang outside the cameras' field of vision. Occasionally we exchange batteries or a light bulb in the boomerang, or adjust one of the field lights. On some nights the sky is cloudy, on others a multitude of stars can be seen. On all nights the grass is very wet. Herman's feet get cold and my left arm becomes tired. Now and then we eat a biscuit and drink a draft of water. On a good night we record over one hundred boomerang flights.(On bad nights, we sit and wait for the wind to vanish, and record none.) When the sky begins to brighten, we disassemble the equipment, put the smaller parts into our suitcase, load everything on the pram, and walk to a little summer house for a few hours sleep.

A substantial part of the boomerang work was neither theoretical nor experimental: it consisted of writing programs and making errors, punching chards and waiting for the automatic plotter to draw computed lift distributions or boomerang flight paths.

Such matters are mentioned hardly or not at all in the three Parts of this report on an investigation into the behaviour of returning boomerangs. Part I presents general information on boomerangs. It contains an ethnographic chapter as well as an elementary explanation of the return flight of boomerangs. It also includes an extensive bibliography. The main research work is reported in the other two Parts. Part II deals with the forces acting on flying boomerangs, i.e. with boomerang aerodynamics. The motion of boomerangs is treated in Part III, which also contains many pictures of theoretical and experimental boomerang flight paths. Both the aerodynamics and the motion of boomerangs have been investigated by means of theoretical models as well as by experiments. Hence, in the boomerang research project four divisions can be distinguished, as indicated in the following scheme:

	theory	experiments
aerodynamics Part II	winglet model	wind tunnel measurements
motion Part III	flight path calculations	field experiments

Each Part begins with an introductory section in which an outline is given of its contents and it ends with a list of references. The Parts have their own numbering of chapters and sections. In the text references are indicated by square brackets. The stereograms of boomerang flight paths can be viewed with the aid of the stereo viewer inserted at the back cover.

§2. *What is new and what is missing?*

Here we give a brief summary of original features of the investigations as well as suggestions for further research on boomerangs.

Part I contains almost no new material. The study of literature on which the ethnographic chapter is based indicates that earlier ethnographers have not observed and reported the flight of Australian Aboriginal boomerangs very accurately. Unfortunately it is too late now to remedy this omission. It is still possible to take detailed measurements of the shapes and mass distributions of Aboriginal boomerangs, with special attention to their aerodynamic properties. A fact too little recognised is that the shape of the cross sections can be more essential to a boomerang's flight than the precise shape of its platform.

Part II, theory. An aerodynamic boomerang model is developed in which the induced velocity of the air is taken into account. Technically speaking: our so-called winglet model is a linearized *pervious* lifting surface theory. The modified, semi-linear version of this model can accomodate boomerang arms with *non-linear* profile lift and drag characteristics (e.g. stall can be taken into account). This model can give no information about the variations of the aerodynamic forces during one spin period of the boomerang. It might be worthwile to develop an unsteady lifting line theory for boomerangs, or adapt an existing theory of this kind to boomerangs. Such a theory should preferably not be restricted to boomerang arms with linear profile lift characteristics.

Part II, experiments. Chapter VI reports measurements of all six force and torque components (averaged over time) acting on rotating boomerangs in a wind tunnel. This appears to be the first instance in which aerodynamic forces on boomerangs have been measured at all. The scientific interest of these experiments may lie in the fact that the local Reynolds numbers of the boomerang arms varied between 0 and 10^5. (Helicopters operate at much higher Reynolds numbers.)

I know of no other measurements of forces acting on rotating wings in a non-axial flow at low Reynolds numbers. The boomerangs used in the experiments were handmade and did not have precisely specified cross sections. The relation between a boomerang's detailed shape and its aerodynamic properties is not obvious. The only investigation I have made concerning this question was done with single boomerang arms in a straight steady airflow (Ch.VI, § 26). However, the results (e.g. with respect to stall) apparently are not valid for *rotating* boomerangs. It would be very interesting to investigate the influence of the shape of the cross sections on the aerodynamic forces acting on a rotating boomerang.

Part III, theory. The equations of motion for boomerangs, which are derived in Chapter I, are "smoothed": variations of physical quantities within one spin period are disregarded by this model. The equations seem to yield satisfactory results, but under one important condition: the boomerang's motion must be stable. It would be of interest to investigate the conditions for the stability of a boomerang's motion. In Chapters III and IV a large number of theoretical flight paths are presented, computed and plotted in great detail.

Part III, experiments. The Chapters II and III deal with experiments in which a lot of boomerang flight paths were recorded: A built-in clock and two cameras were used, so that, in principle, the boomerang's position as a function of time could be determined. A fundamental weakness in these experiments is that the initial conditions of the flights are not precisely known. For better field experiments the development of an accurate boomerang-throwing machine would be essential. On the other hand, one need not have access to laboratories or computers in order to contribute to the knowledge on boomerangs. Serious hobbyists might obtain significant results by experimenting with carefully made boomerangs of precisely known shapes.

§3. *Acknowledgements.*

I am grateful to many people who contributed to the work which is reported in the following three Parts. Prof.Dr. J.A. Sparenberg thought the subject of boomerangs worth of serious investigation, and he introduced me to the field of fluid dynamics and lifting surface theory. His openmindedness to unorthodox approaches stimulated the development of the winglet model.

Three series of laboratory experiments have been carried out. The first experiments, with metal boomerangs under water, were carefully executed in 1970 in the Shipbuilding Laboratory of the University of Technology Delft by Ir. M.C. Meijer and Mr. A. Goeman. The object of these measurements was to check the theoretical results of the winglet model. The second series of experiments concerned boomerangs in a straight airflow. The measurements were carried out in December 1970 at the Twente University of Technology with the help and advice of Mr. G.H.M. ter Bogt and Dr.Ir. H.J. van Oord. My wife Wietske assisted me in operating the small wind tunnel. The third and most important series of measurements was carried out in the weeks around Christmas 1971, again in Delft. Forces on rotating boomerangs were measured in a wind tunnel of the Laboratory for Aero- and Hydrodynamics. Much help and advice was given by Ir. H. Leijdens and Ir. R.E. de Haan of this lab. The measuring elements and the electronics were supplied by the Shipbuilding Laboratory. Ir. M.C. Meijer offered a lot of expert advice. Mr. A. Goeman worked with unbelievable diligence and accuracy, operating the electronics and writing down thousands of numbers. Some equipment for each of the three series of experiments was made in the workshop of the Laboratory for General Physics in Groningen. Herman D. Coster assisted me in the design of this equipment. His combination of technical insight and fundamental thinking can be recognized in such features as the tetrahedral frames and the phosphorbronze wires of the measuring apparatus used in the wind tunnel experiments (Part II, §27).

The field experiments in which boomerang flight paths were photographically recorded took place near Steenwijk in the summer of 1973. Herman Coster operated the cameras, manufactured the surprisingly simple field lights, suggested the needle-on-glass bearings for the wind meters and designed the "time pill" (Part III, § 11). This is a tiny electronic device serving as a clock, we put it together in three weeks of very careful work. One of the cameras was kindly lent by Bob Kaper. Hans Rollema and Wietske assisted me in some earlier field experiments.

The ethnographic Chapter in Part I is written by a layman: I am not a cultural anthropologist. It was fortunate that several Australian anthropologists answered my letters and offered me useful suggestions. Especially Mr. Frederick D. McCarthy (Sydney) and Dr. A.C. van der Leeden (Nijmegen) gave me valuable advice. Much information on the prehistoric Velsen boomerang was given by its finder, Mr. A.J. Schotman of the Royal Dutch Blast Furnaces and Steelworks. Mr. B.A.L. Cranstone of the British Museum (London) kindly helped me to investigate an Australian cross boomerang. Many articles on boomerangs were brought to my attention by Mr. Benjamin Ruhe of the Smithsonian Institution (Washington,D.C.). Probably there is no boomerang activity anywhere in the world without Ben Ruhe knowing about it.

The way from scientific curiosity to scientific report is a long one. An essential role in much of the boomerang work was played by electronic computers.(Once addicted to their use, one cannot do without them.) In the course of seven years many theoretical boomerangs were computed and numerous flight paths plotted, first on the TR4 and more recently on the Cyber 74-16. The people of the Computing Center of the Groningen University always offered expert assistance in a supple way. The Photographic and Audiovisual Departments of the university have processed an amazing amount of photographs and drawings. One difficult photographic job was done by Mr. H. Leertouwer of the physics lab.

Greet Boerema typed nearly all of the manuscript. Many of the drawings were made by Bob Kooi. Jos Hess assembled the stereo photographs. Frank Venema printed a text on the stereo viewers. In the final hectic weeks Trientje Stuit and Trudy Klosse assisted in the typework, Bob K. drew lines and numbers during "afternoons" which ended at 10 p.m., and Jos H.,Berber W. and Wietske pasted in hundreds of illustrations.

Prof. Sparenberg critically read all of the manuscript. Prof.Dr.H.de Waard carefully read the experimental chapters. Mr. F.D.McCarthy and Dr. A.C. van der Leeden kindly read the ethnographic chapter.

The work reported in Part II, Chapter I through IV was financially supported by the Netherlands Organization for the Advancement of Pure Research, Z.W.O.

There are still others who contributed to the pleasure of studying boomerangs. In June 1973 Herman and I visited Dr. Peter Musgrove at the University of Reading (England). We exchanged a lot of information about boomerangs with Peter and witnessed the boomerang launcher built by his students. There we met Allan Grantham who gave me a particularly good left-handed boomerang of his own make, Vivian Davies, the egyptologist from Oxford who informed me about Tut Ankh Amen's boomerangs, and Major Christopher Robinson, the secretary of the Society for the Promotion and Avoidance of Boomerangs.

Dr. E. Whittaker (Dapto. N.S.W. Australia) sent me a beautiful left-handed Aboriginal boomerang, which can return perfectly. Mr. Willi Urban (Leutershausen, Germany) sent me his excellent "come back", and wisely warned me against attempting to write a doctor's thesis on boomerangs. Dr. Russell B. Snyder (Eldorado, Kansas) wrote many a cordial letter and especially made me nearly a dozen left-handed boomerangs. Mr. Herb A. Smith (Arundel, Sussex) made me two left-handed very-long-distance boomerangs and supplied a sketch of his record boomerang throw. Then there are Claude Zeyen (Luxembourg),Gordon Rayner (San Diego, Calif.), Dr. Lorin Hawes (Mudgeeraba, Queensl. Austr.), Roger Luebbers (Canberra, Austr.) and many others with whom I have exchanged information on boomerangs.

The seven years spent on this project have been much more than I had expected to be necessary for learning "all about boomerangs". But still I do not know such things as: how to make a boomerang that with little effort can be thrown extremely far and return perfectly without the help of wind.

Groningen, May 1975,

Felix Hess

PART I
GENERAL

So far the civilized uses of this ethnological curiosity have been confined to abstruse mathematical calculations of its complicated movements, while it furnishes a dangerous plaything for mischievous school-boys.

[Parry, 1872, p. 400]

§1. *Introduction*

Part I of this work contains a great deal of background information on boomerangs, but presents little new material. Chapter I gives information related to ethnography, cultural anthropology and archaeology. This chapter deals with such questions as: Where, inside and outside Australia, have boomerangs been found? How are boomerangs manufactured and used by the Australian Aborigines, and for what purposes? Nearly all of the information collected by me on these topics is based on literature rather than on own observations. It is second-hand knowledge the validity of which entirely depends on the selection and the quality of the sources used. The choice of the topics, with a strong emphasis on the aspects of boomerangs most directly related to their physical properties, reflects of course, my own background in physics and mathematics. Such matters as the more general cultural functioning of boomerangs in Aboriginal Australia are barely mentioned. Within these limitations Chapter I offers a fairly complete survey of what is known at present about boomerangs in cultures other than the "modern western civilisation."

Chapter II gives information related to mechanics, physics and mathematics. It presents an explanation at an elementary level of the return flight of boomerangs, and provides the reader with a basic understanding of the mechanics and aerodynamics of boomerangs. A short survey is given of the physical and mathematical research on boomerangs from 1837 up to 1975.

Finally, Chapter III consists of an extensive bibliography on boomerangs, which contains nearly 400 items. The nature and subject matter of each item is indicated. Though certainly incomplete, this probably is the most comprehensive bibliography on boomerangs to date.

CHAPTER I

BOOMERANGS FROM AN ETHNOGRAPHICAL VIEWPOINT.

§2 *Various types of Australian boomerangs.*

As is well known, boomerangs are used and manufactured by the Australian Aborigines. The term "boomerang" generally is rather vaguely defined: a wooden object which can be thrown in such a way that it rotates rapidly and, by its interaction with the air, traverses a flight path which differs considerably from that of an ordinary thrown stick.

This curious and unique weapon, about which so much has been written and so little is really known, is a curved piece of wood, slightly convex on one side and nearly flat on the other. It is cut from a natural bend or root of a tree, the hardest and heaviest wood being always selected, and its curve follows the grain of the wood. Thus it will vary from a slight curve to nearly a right angle; no two ever being the same shape. It is about three-eights of an inch [1 cm.] thick, and from two to three inches [5-7½ cm.] wide, tapering toward the ends, which are either round or pointed. The edge is sharpened all around, and the length varies from fifteen inches to three and a half feet [40 cm.-1 m.]. [Baker, 1890, p. 375].

Most Australian boomerangs do not return to the thrower. According to Davidson [1935b, p. 163]:

Of the popular fallacies associated with the boomerang perhaps the most widespread and deep-rooted is the belief that all boomerangs are of the returning type. As a matter of fact, returning boomerangs constitute only a very small percentage of Australian boomerangs, a percentage difficult to estimate accurately but which under normal aboriginal conditions may have been exceedingly small.

Non-returning boomerangs, also called war, fighting or hunting boomerangs, can fly along a more or less straight horizontal line for long distances, and strike an object, prey or enemy with amazing force. According to Blackman [1903, p. 48]:

The war boomerang is an effective and dangerous weapon, having a range of 150 yds. [140 m.], and having been known to pass completely through an adversary when the body was first struck by the point of the weapon.

Returning boomerangs are mainly used for play; Davidson [1935b, p. 163/4] says:

It is also generally believed that the returning boomerang is a weapon and that it is used in war and for hunting. These functions, however, with few exceptions, are found associated only with ordinary boomerangs. The returning boomerang is regarded by the aborigines as a toy to be thrown for amusement. In only a few instances are there reports that it is used for other purposes, although in emergencies it may be utilized as a weapon in the same manner as any other suitable object.

Davidson [1935b, p. 164] continues:

It is likewise important to understand that the returning boomerang, contrary to popular belief, will not return to the thrower if it strikes any object during its flight. In most cases such a happening would cause it to fall directly to the ground. Occasionally if an obstacle is not struck squarely, the stick may be deflected and started on a different course of flight, but in such an event, the point of landing would be altered. The common belief, therefore, that the boomerang will return to the hand of the thrower after it has struck the enemy or the prey has no basis in fact.

In addition to returning and non-returning boomerangs the Australian Aborigines use also a great variety of other wooden throwing or striking implements, ranging from straight sticks and clubs to boomerang-like objects with one very broad, flat end, called Lil-lil [Foy, 1913; Davidson, 1936; Smyth, 1878; Etheridge, 1897b; Sarg, 1911], and there is a multitude of intermediate forms. As to the variations in boomerangs Davidson [1935b, p. 165/6] remarks:

Boomerangs vary so much in their forms, sizes and weights that it is a difficult matter to classify them into types and varieties. Some are symmetrical in form, others have one arm longer than the other. In many specimens the width is fairly constant throughout the greater part of the weapon, in others it is relatively great at the bend and may decrease gradually or abruptly as the ends are approached. The degree of curvature also shows much variation and may range from a right angle or less to almost 180°. For the shape of cross-section, we find some examples thin and wafer-like, whereas others may be almost round, or in those specimens having one flat surface, almost hemispherical. The extremities run the gamut from round to pointed, and in addition there are several varieties of specialized ends with angular or other features which set them off from the more usual forms. Finally there is the question of a longitudinal twist, a feature necessary to the returning boomerangs but also one occasionally found to a slighter degree and perhaps accidentally in those not intended for use as playthings. When all these variable features are taken into consideration it is clear that the number of combinations is infinite and that it is a most difficult matter to describe the differences between the boomerangs of many regions of the continent. There is no one feature sufficiently constant to serve as a standard.

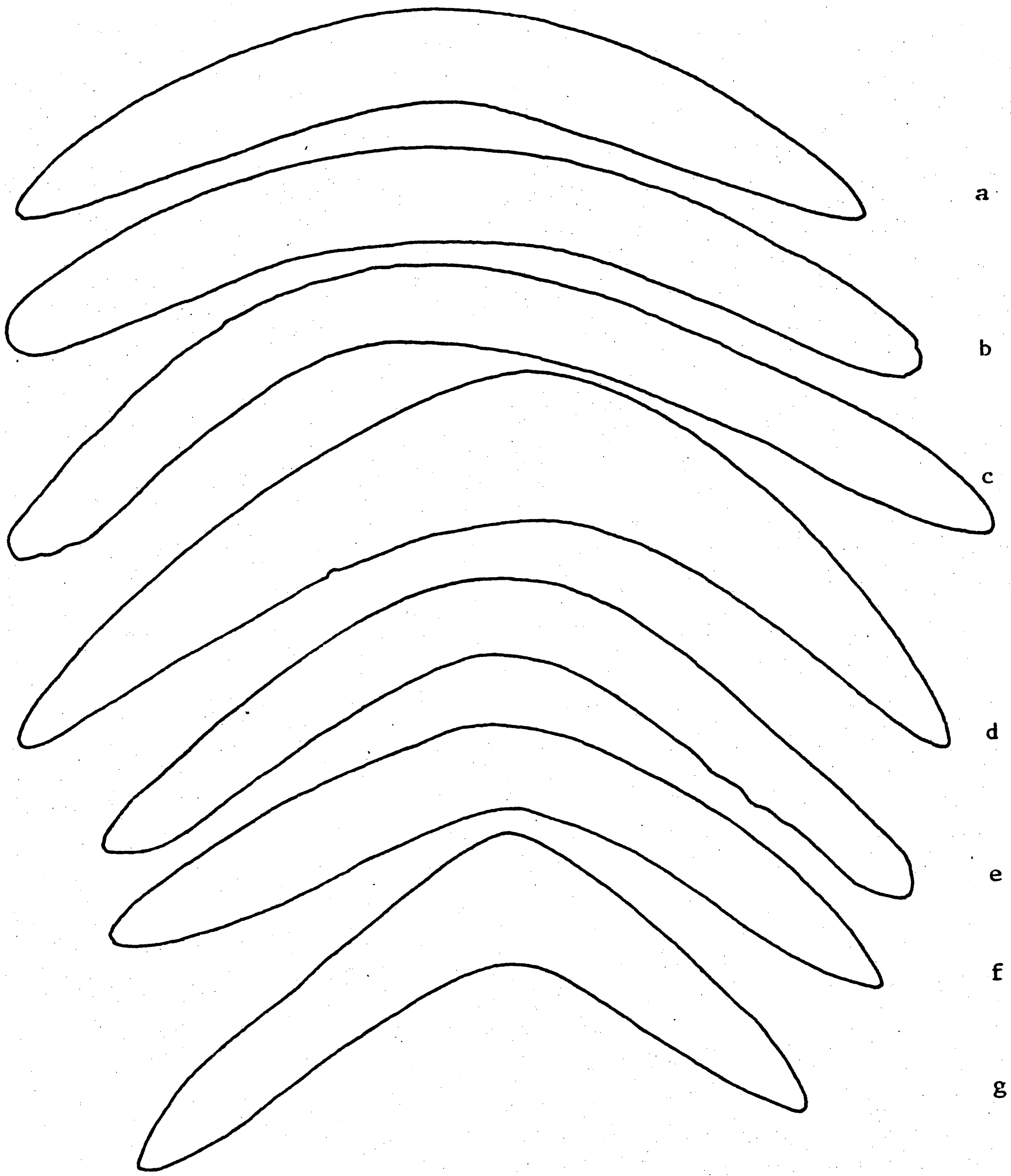


fig. 2.1 Planforms of some returning boomerangs. After [McCarthy, 1957, p. 80].
a: Fitzroy river, Western Australia. b: Murchison river district, Western Australia. c: eastern New South Wales. d: King Sound, Western Australia. e: Kimberleys, Western Australia. f: Western Australia. g: Kimberleys, Western Australia.

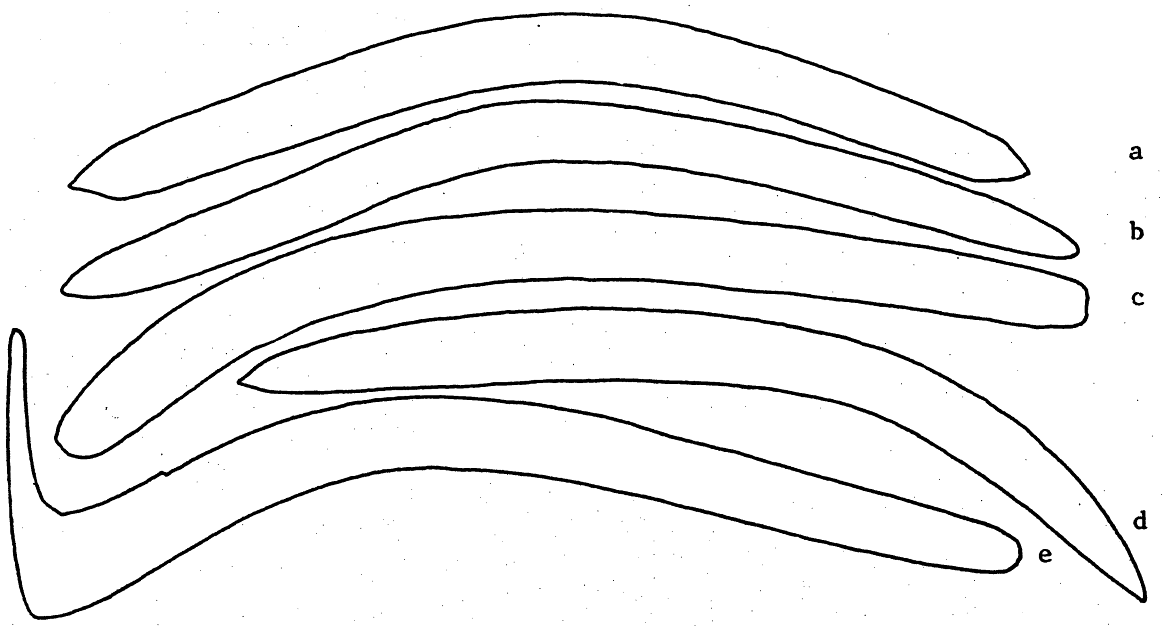


fig. 2.2 Planforms of some non-returning boomerangs. After [McCarthy, 1957, p. 78]

a: Cloncurry, Queensland. b: Bogan river, New South Wales.

c: Central Australia. d: Rockhampton, Queensland.

e: Northern Territory, hooked boomerang.

It is often difficult to determine whether a particular boomerang is of the returning kind or not, by just inspecting its shape. To make matters more complicated:

There are many left-handed native boomerangs. These, when thrown with the right hand, often go straight away, and do not return. [Pern, 1928, p. 103].

Thus, according to Smyth [1878, p. 318]:

One can easily imagine the perplexity of an enquirer who should have a number of these instruments presented to him, some left-hand, some right-hand, and some apparently of the like form, but not made to return. His experiments with them would but embarrass him the more; and if he succeeded in throwing one weapon successfully again and again, he might conclude that his want of success with the others was due solely to their imperfections.

McCarthy [1957, p. 97] says about returning boomerangs:

They are usually similar in style to the local non-returning forms but are smaller, thinner, and more deeply curved.

Roth [1897, p. 128], referring to North West Central Queensland, states:

The "Come-back" or "Return" Boomerang [...] is a toy which, compared with a fighting boomerang, is always lighter, much smaller, and varies in shape from a comparatively strong angle to something approaching a half-moon, the knee or bend being always in the centre. Sometimes it is cut down from one of the other kinds of boomerang that has been damaged or broken.

Smyth [1878, p. 317], comparing boomerangs from Victoria, says:

The most obvious difference of form between the boomerang which returns and that which does not return is in the curve, looking at the flat side of the weapon [...]. The *Wonguim* [returning] exhibits almost invariably a much sharper curve than the *Barn-geet* [non-returning]; ...

A Wonguim including an angle as sharp as 70° between its arms is represented in [Sarg, 1911, p. 12].

Howitt [1876, p. 248], also writing about Victoria, reports:

Two kinds of boomerang are made, one called "marndwullun wunkun," that is the "boomerang," as I may translate the term "wunkun," which turns round; "marndwullun" is equally applied to the returning flight of a bird as to a boomerang. The second kind of boomerang is called "tootgundy wunkun," that is the boomerang which goes straight on, "toot" meaning something "straight" or "erect."
The two boomerangs differ in their construction. The second (straight) kind being thicker, longer, and less curved than the first, I shall call, as a matter of convenience, the "marndwullun" No. 1, and the "tootgundy" No. 2.
With No. 1 there is no certainty of hitting the mark. It may come back too quickly, and may hit your own friends standing near you. In choosing a boomerang like No. 2, in preference, it will be more sure to hit the object, ...

McCarthy [1957, p. 88] gives the following descriptions:

The non-returning boomerang is a crescent from two to three feet [60-90 cm.] long, possessing a shallow curve in relation to its length, and weighing up to one and a half pounds [700 g.]. [...]
The returning boomerang is a much smaller weapon than the fighting type. It is a thin and well-balanced missile from one to two feet six inches [30-75 cm.] long and up to twelve ounces [350 g.] in weight.

Cf. [Smyth, 1878, p. 311]:

The weight of these weapons varies from four ounces to ten and a half ounces [110-300 g.]. Those as light as four ounces are rarely used in

Victoria, but such light weapons seem to be much in favor in Western Australia.

McCarthy [1957, p. 88] continues:

It is always deeply curved, and some specimens have two distinct arms with a sharply angled bend. Both sides may be convex or one convex and the other flat. The weapon's outstanding characteristic is that one end is twisted upwards and the other downwards from front to back, in a contra manner. This is usually done by soaking in water and then heating over a large fire or in hot ashes until it is pliable enough to twist.

This *twist* is considered essential to the return behaviour of boomerangs by many authors, and we will consider this question in §4.

Intermediate types, between returning and non-returning boomerangs, apparently also exist. According to McCarthy [1957, p. 97]:

Several types of boomerang, notably the *kaili* plano-convex Mulga wood type of Western Australia, and the *bou-ma-rang* (as it was originally named from the Turuwal language on the George's river, near Sydney), a bi-convex mangrove wood type from eastern New South Wales, probably served as dual types, being heavy enough for use as hunting and fighting boomerangs, and not too big to serve as returning boomerangs. The twist could be added or eliminated at any time, as the occasion necessitated.

A special type of boomerang is the hooked, horned, beaked or swan-necked boomerang (see fig. 2.2e). It is described by Roth [1897, p. 145/6] as follows:

It differs from the fluted boomerang in the possession of a hook [..], from 4 to 5 inches [10-13 cm.] long projecting backwards, in the same plane, from the extremity of the shaft on the convex edge: this hook, about an inch [2½ cm.] or more wide at its base, tapers gradually to a blunt point, and bears a longitudinal fluting continuous with that on the main shaft. Furthermore, the shape of the shaft contrasts markedly with that of all other boomerangs in its width, independently of the bend or knee (*not* its widest part), increasing progressively from the proximal to the hooked extremity.

Balfour [1901, p. 33] describes an unusual variety from MacArthur River:

Instead of being cut out of a single piece of wood specially selected for the purpose, as is the case with the swan-necked boomerang as usually seen [..], this example has been apparently made from an ordinary boomerang having but slight curvature, and the spur at the end is formed by fixing with gum a flat piece of wood to the boomerang head. The spur is painted in red and white patterns, and the boomerang is coated with red ochre. The spur is protected with a sheath of *melaleuca* bark. The hook-like spur is 6½ inches [16½ cm.] long.

The function of the hook seems uncertain. McCarthy [1961, p. 348] gives as his opinion:

The hooked or swan-necked variety of the fluted boomerang in the Northern Territory is a well-balanced boomerang. It is said that the hook catches on the edge of a shield or spearthrower (used for parrying) and the shaft whips round and strikes the defender. This action could not be very dangerous, and I believe that the hook forms a pick for fighting at close quarters, like the stone-bladed pick in this region.

A peculiar form of boomerang is used as a toy in the region around Cairns (Queensland). An early discription of it is given by Roth [1902a, p. 513] = [1902b, p. 19]:

The *"Cross"* is made of two pointed laths, from about 8 to 10 or more inches [20-25 cm.] long, drilled at their centres and fixed cross-wise in position with split lawyer-cane [...]. It is met with in the coastal districts extending from Cardwell to the Mossman, and to the Mallanpara blacks of the Tully is known as pirbu-pirbu.

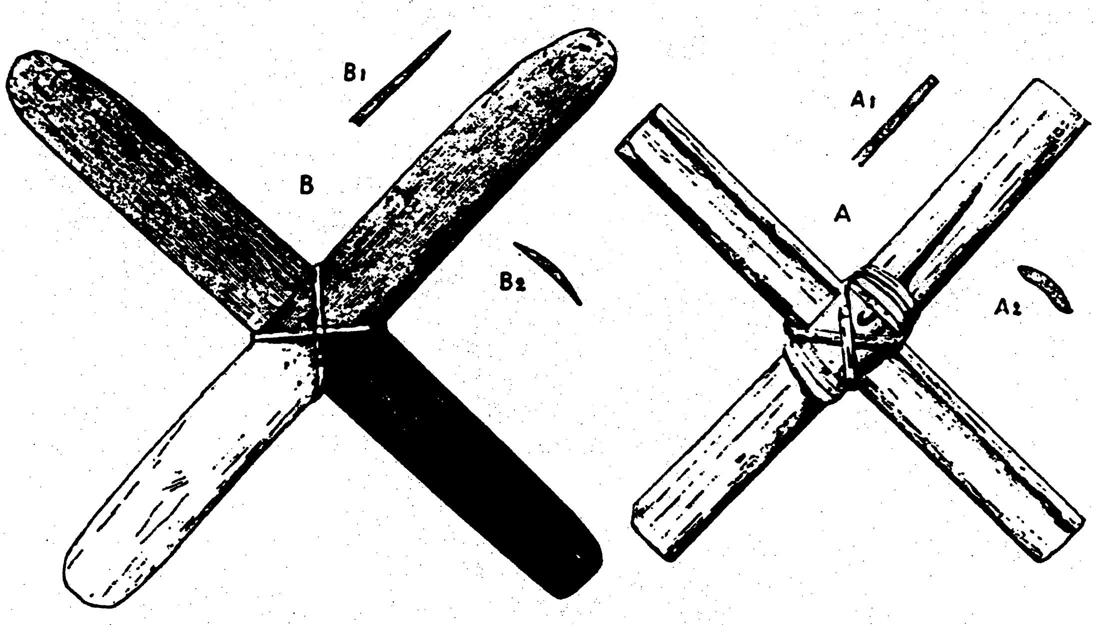


fig. 2.3 (copied from [Kaudern, 1929, p. 236]) Cross boomerangs.

B from Queensland, A from Celebes.

B1, A1 sections in spanwise direction.

B2, A2 cross sections in chordwise direction.

Kaudern [1929, p. 236/7] describes such a cross boomerang in connection with a similar one from Celebes (see also §9):

This cross is made from some light wood of light colour. The limbs are respectively 36,5 cm. and 36 cm. by 4 cm. They are only 5 mm. thick. The ends are rounded off and the limbs all round trimmed down to an edge. The outside of the limbs is slightly convex, the inside concave similarly to the boomerang from Ondae [Celebes]. The limbs are lashed together with a strip of rattan in the same way as the Ondae cross [...]. In the Australian boomerang two of the limbs are painted with red, one is black and one is white.

It is remarkable that with both the Celebes and the Queensland cross boomerang described by Kaudern the limbs are fastened with the concave sides against one another. This can be clearly seen in the accompanying illustration (fig. 2.3). Thus one limb would be aerodynamically upside down during flight, which would seem to have adverse effects on a boomerang's flying behaviour. None of the other references I could find in the literature contains information on cross sections of the limbs of cross boomerangs. The one cross boomerang I scrutinized (see fig. 2.4) has limbs with symmetrical biconvex cross sections, and without twist.

Cross boomerangs can return quite well:

..., thrown direct into the air, the course of flight is similar to the boomerang, but there is more of the circle than the oval, and a *double* circle round the player at its termination. [Roth, 1902, p. 513].

Pern [1928, p. 101] relates:

I have some toy boomerangs [...], which came from the Cardwell district, south of Cairns. They are of various shapes, and amongst them is one made by crossing two pieces of reed. These cross boomerangs, when thrown at the right angle and height, will return, but have not the same liveliness of flight as have the others. Curiously enough, they will go equally as well with one or even two blades off, as long as they are at right angles to one another.

A variety is mentioned by Roth [1902a, p. 513] = [1902b, p. 19]:

The above toy is imitated by some of the smaller children by means of thick swamp-grass, &c. The two strips are either pierced and tied, as in the case of the wooden ones, or else plaited together [...]. It is thrown with a twist of the wrist up into the air, whence it soon returns in a right or left spiral.

Pictures of cross boomerangs are given by Davidson [1937, p. 28] (copied in [Guiart, 1948, p. 30]), Sarg [1911, p. 13], Kaudern [1929, p. 236], Kennedy [1949, p.21], McConnel [1935, p.62,67], Cranstone [1973, p.16].

Australian boomerangs can be decorated by painting and by incising. A detailed account of the decorations of boomerangs is given by Davidson [1937, p. 18-28]. Since our main interest in boomerangs concerns the mechanical and aerodynamic properties, the decorations may be left out of consideration here, except for one point. The fluting (parallel grooves) on the convex side of the fluted boomerang might be aerodynamically relevant. Hillyer [1909, p. 256] suggests:

The flutings, like the furrows on a golf ball, are to make the boomerang "bite" the air ...

Indeed the fluting might reduce the air resistance of the boomerang arms. See also §4.

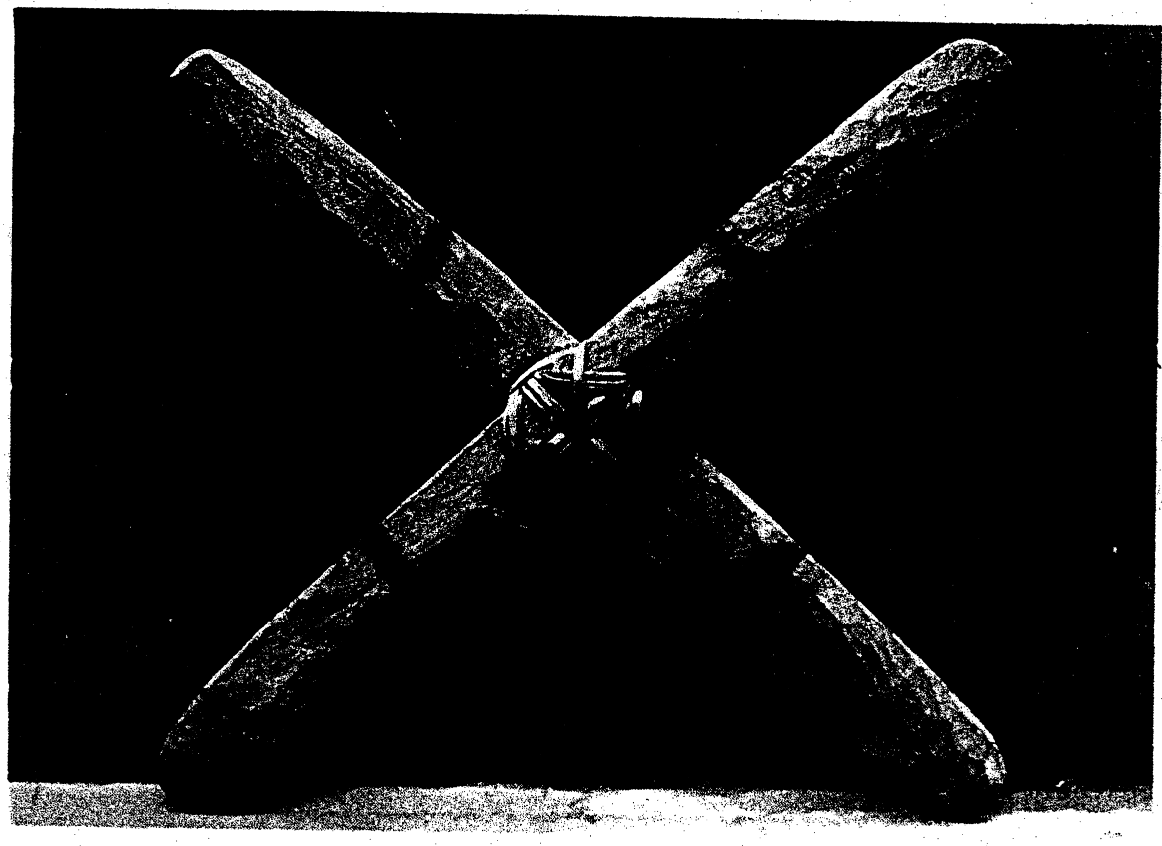


fig. 2.4 Cairns cross boomerang. (Museum of Mankind, British Museum) Shown from the other side in [Cranstone, 1973, p. 16]. Roughly chipped from light wood. Weight: 73.5 g. Length of limbs: 37.2 resp. 36.4 cm., greatest width: 4.1 resp. 4.5 cm. Thickness: between 0.8 and 1.1 cm. Thickness/chord fairly constant: 0.23-0.26. Biconvex symmetrical profiles. Measured and photographed with permission of Mr. B.A.L. Cranstone.

§3 *Distribution of boomerangs in Australia.*

The best source of information on the distribution of boomerangs in Australia appears to be Davidson [1936], who also gives quite some references. According to him [1936, p. 88-90]:

> Boomerangs, as a class, are widely distributed in Australia but are not continental. It is important to note that they are lacking in Tasmania and in all the northern peninsulas of Australia, the Kimberley coastal country, Groote Island, and North Australia approximately north of a line drawn from the Katherine River to the Roper River, and in Cape York peninsula, north of the Mitchell and Palmer Rivers [...]. There are also a few minor districts in which they seem to be unknown.

For example, Tindale [1925, p. 99] mentions about the Ingura on Groote Eylandt:

> The boomerang [...] is known to them only from exaggerated rumours of their wunderful killing power.

As regards the distribution of returning boomerangs Davidson [1936, p. 97] gives the following account:

> Returning boomerangs are widely distributed but are not found in all regions in which the ordinary forms appear [...]. There seems to be no indication that the returning kind is ever present by itself. The major regions in which returners are or were used include at least parts of Victoria, New South Wales, Queensland, South Australia, and Western Australia. The main negative area, aside from those in which no boomerangs at all are present, is the Central Australia-North Australia region. We thus find that returning boomerangs, like incised boomerangs, occupy an area which almost surrounds an area in which they are not used, but where the so-called "fighting" boomerang is present. Since the latter is definitely known to be diffusing outward into areas where the former is now found, the question arises as to whether returning boomerangs formerly occupied a wider distribution in a part of the region in which we now notice only the so-called fighting type. This question cannot be answered at present.

According to McCarthy (personal communication, 1974) an aspect worth considering is:

> the boomerang in trade as this has an important bearing on the distribution of the types and their uses, e.g., clapsticks in Arnhem Land for traded boomerangs, and a rock engraving of a hooked boomerang at Port Hedland hundreds of miles from where it was made.

Maps showing the distribution of boomerangs in Australia have been given by Davidson [1936, p. 89], [1937, p. 19], Guiart [1948, p. 33] and McCarthy [1961, p. 344]. Precise boundaries of non-use of boomerangs in

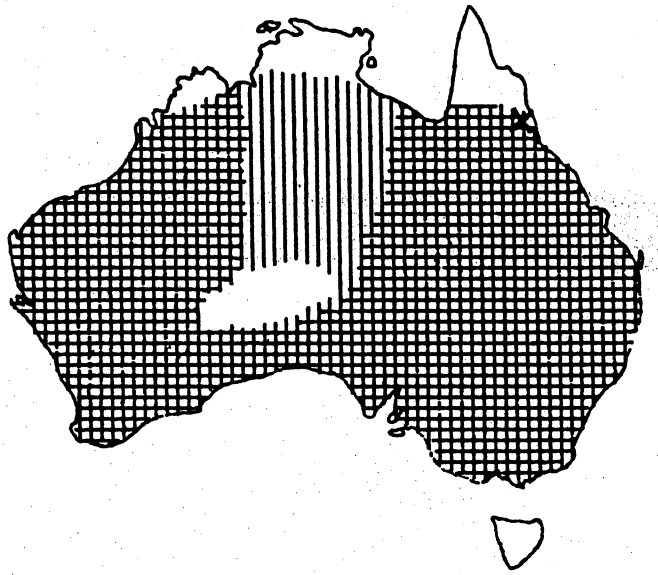
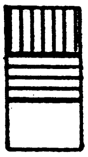


fig. 3.1. Distribution of boomerangs in Australia. (After [McCarthy, 1961, p. 344] and [Davidson, 1936]).



non-returning boomerangs

returning boomerangs

boomerangs not made (received by trade in some areas).

× cross boomerangs.

the Western Desert area are not known [McCarthy, personal communication, 1974].

§4 *The cross section and twist of Australian boomerangs.*

The cross section of a boomerang, rather than its planform, is of crucial importance to its flying behaviour [Hess, 1968a], see §16. This has been noted by many investigators. For instance, Smyth [1878, p. 318] states:

The form of the weapon in section is apparently essential to its flight and return. It is observable in all the specimens I have examined, and in all, whether right-hand or left-hand, the flat side in gyration is towards the earth.

It is, however, a curious fact that in most literature on Australian boomerangs very little attention is paid to the various shapes of boomerang cross sections. Only in a few instances are drawings of such cross sections presented, and then almost exclusively sections through the central part of a boomerang, e.g. [Smyth, 1878, p. 318]. The only favourable exception I am aware of is Turck [1952], who gives drawings of 17 Australian boomerangs with at least three cross sections taken at different points of each boomerang. He also gives some detailed measurements and mentions the region of origin of each boomerang. Although this is not mentioned by Turck, probably the boomerangs studied by him are museum specimens, which may have become warped.

It appears from the available literature that most Australian boomerangs have a lens-shaped cross section throughout their whole length, and that often one side is more convex than the other side, which may even be flat. The shape of the cross section generally has a front-back symmetry, i.e. the maximum thickness is in the middle of the profile, and there is no difference between the leading and the trailing edge of a boomerang arm. If both arms would be kept in exactly the same plane, the boomerang's aerodynamic properties would be independent of the sense of rotation imparted to it. In other words, it could be used both as a right-handed *and* as a left-handed boomerang. In reality, however, most boomerangs are somewhat twisted, either accidentally or on purpose. This probably would make them suitable to fly reasonably well with one particular sense of rotation only, hence they would be either right-handed *or* left-handed boomerangs. On this point Waite [1930, p. 436] remarks:

Scissors, as ordinarily constructed, naturally cannot be used by left-handed people and, similarly, a left-handed thrower cannot use a normally right-handed boomerang; weapons with reversed inclination are known, and indicate use by left-handed throwers.

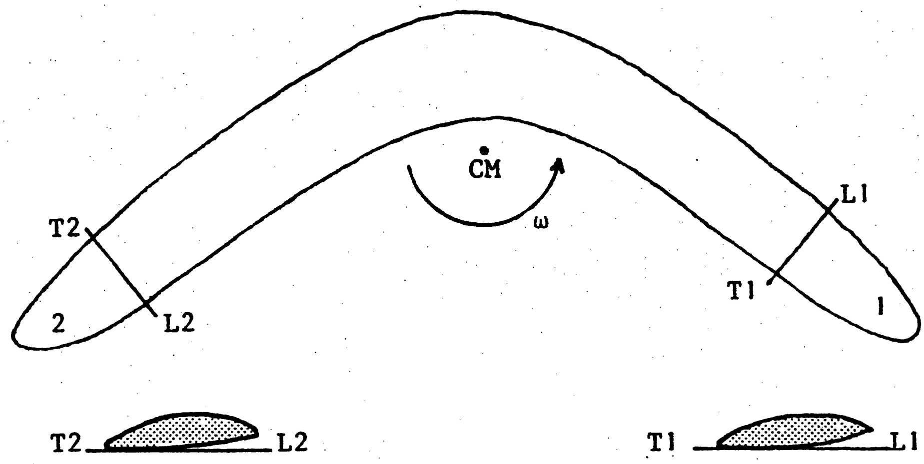


fig. 4.1. Sketch of a boomerang with twist (right-handed).

L1-T1 and L2-T2: cross sections through boomerang arms.
L1, L2: leading edges. T1, T2: trailing edges.
CM: centre of mass of boomerang. Arrow ω: sense of rotation.

A right-handed boomerang designed with an intentional twist is sketched in fig. 4.1. If such a boomerang is placed on a plane support, its more convex side uppermost, the leading edges of its arms (L1, L2) would be raised above the supporting plane. (A similar description is given by Thomas [1910, p. 236].) It is this *twist*, also called skew (Drall in German), which is claimed by many authors to be essential to the return flight of boomerangs. For instance, Oldfield [1865, p. 265] states:

The efficacy of the boomerang does not at all depend on the inclination of its arms to each other, its whole efficiency being due to a twist by which the plane of one arm is made to depart from the plane of the other, just as the sails of a windmill are not in the same plane.

Other authors who state that the twist is an *essential* characteristic of returning boomerangs are:Erdmann [1869], Stille [1872], Smyth [1878], Hardman [1886], Lumholtz [1889], Buchner [1905, 1916, 1918], Thomas [1910, 1955], Sarg [1911], Sutton [1912, 1939], Thorpe [1924, 1926], Nevermann [1925], Bonnet [1926], Franz [1928], Davidson [1935b], McCarthy [1957, 1958a, 1961, 1965]. Erdmann and Stille considered boomerangs from a physical viewpoint, and the twist ("windschiefe Fläche") is an essential feature in their theoretical models. Probably most of the other authors relied on earlier sources rather than assessing themselves the function of twist in boomerangs. There are authors who state that twist causes what might be called a "helicopter effect" in boomerangs. For instance, Lane Fox [1868, p. 425] mentions:

> ... a slight lateral twist, by means of which it is caused to rise by virtue of its rotation, screwing itself up in the air ...

(See also [Lane Fox, 1877, p. 30].) And Smyth [1878, p. 318] states:

> This twist is the twist of the screw, and the property the boomerang has of ascending is due to its having this form.

This "helicopter effect" is indeed of importance to boomerangs, but it does not necessarily require the presence of twist (see §16). Boomerangs without any twist may perform very well and return perfectly, as I know from my own experience. This is also reported by Hillyer [1909, p. 256], and Salet [1903, p. 186] even asserts:

> J'ai trouvé ainsi que la face inférieure du boomarang doit être absolument plane ...

But, although twist may not be a *necessary* feature of returning boomerangs, there can be little doubt that many Australian returning boomerangs possess a certain amount of twist of the kind indicated in fig. 4.1. For instance, Smyth [1878, p. 322] says:

> I never saw a *Wonguim* made by the natives of Victoria which was not twisted. The thin leaf-like weapons of the West Australians are twisted. In some the twist is so slight as to be scarcely perceptible, but it is there, and can always be discovered.

The last sentence of this quotation would suggest that the presence of twist in some Western Australian returning boomerangs (kylies) can only be discovered if one is purposely looking for it. More explicitly Smyth [1878, p. 335/6] remarks about kylies:

> At first sight they appear to be quite flat; but a close examination shows that there is a slight twist; and in weapons so thin as these a very small deflection is sufficient to ensure their true flight and their return to the thrower.

As in such thin implements even a slight twist would be relatively easy to detect, this passage raises some doubt as to the universality of twist in Australian returning boomerangs. It is probably too late now to investigate this matter in the field, and boomerangs kept in museums may have become warped so that their original twist cannot be assessed anymore:

> As the result of warping it is often impossible to distinguish returners from ordinary boomerangs in museum collections. [Davidson, 1936, p. 97]

It should be clearly understood that if a given boomerang with good flight properties suffers a change in the amount of twist, either intentionally or accidentally (warp), its behaviour in flight may be changed considerably. (See also §5).

Non-returning boomerangs may have a negative twist, i.e. a twist opposite to the one indicated in fig. 4.1. This has been remarked by Walker [1901a, p. 457/8] = [1901b, p. 338] = [1901c, p. 516] and also by Thomas [1910, p. 236] = [1955, p. 883], Carter [1933, p. 142] and Musgrove [1974, p. 188]. Indeed, a certain amount of negative twist may allow a boomerang to traverse an approximately straight flight path. See [Musgrove, 1974, p. 188] and §17.

Finally, it may be remarked that small-scale features of a boomerang's surface may influence its flying behaviour. The presence of dimples or ridges (fluting) might reduce the air resistance, as was already noticed at the end of §2. See also similar remarks by Moulder [1962, p. 59] and Wood [1974]. The sharpness or bluntness of the leading edges and the smoothness or roughness of the surface might also be aerodynamically relevant (see §17). However, these matters have not been investigated as yet. McCarthy (personal communication, 1974) remarks that the aerodynamic effect of fluting on boomerangs would be accidental or fortuitous, as similar fluting is also applied to shields, wooden containers, clubs, throwing sticks, where it has a decorative function.

§5 *The manufacturing of Australian boomerangs.*

There exists a 16 mm. documentary [Campbell, 1958] showing in detail how a boomerang is manufactured by an Australian Aborigine. In this case the Aboriginal craftsman Yaningi Djungerai made a fluted (non-returning) boomerang near Yuendumu, Central Australia in 1958. The film shows that he carefully selected a mulga tree which he cut down at shoulder height. This was done with a "European" axe, but in former times stone axes were used. From the fallen trunk he cut off a curved piece of about 1 m. length and 15 cm. diameter. This log was fixed in an upright position, a hole in the ground being used as a vice. The log then was cleaved in two, and the thinner part was discarded. Four or five notches were made in the round side of the other part, so as to control the splitting of the wood. Then pieces of wood were chopped off with the axe, leaving a flat board of about 2½ cm. thickness with a curved planform. The edges of this board were trimmed with the axe so that a rough boomerang planform resulted with a length of about 75 cm. From this point on the craftsman used various stone chisels. Gradually the piece of wood acquired the desired shape. Every now and then the craftsman critically viewed the object from different directions. When the boomerang looked almost finished, it was rubbed with sand. With a very fine chisel the ornamental fluting was made on the more convex side of the boomerang: some 13 parallel grooves. Each groove proceeded first from a point about 7 cm. from the tip of one arm to the "elbow" of the boomerang and then from the other arm to exactly the same point on the "elbow". Finally the craftsman applied a coat to the whole surface of the boomerang by rubbing it with wet red ochre.

The manufacturing of boomerangs by Aborigines has been described by many authors. A rather detailed account, as far as North West Central Queensland is concerned, is given by Roth [1897]. In his sec. 151 [p. 102] he treats the artificial bending and straightening of timber:

The aboriginals throughout all the different ethnographical districts both know and practise various methods of bending or straightening timber, either when already cut or in the rough. Thus, a dry heat in ordinary sand, a moist heat from burning freshly-gathered gum-leaves, or moisture in general, such as soaking in water, is employed for bending any of their wooden implements into shape as required. In order to maintain and preserve the timber in the position attained by one or other of the preceding processes, the whole is covered thickly with grease and fat, saurian of mammalian.

On the making of boomerangs in particular Roth [1897, p. 142] says:

The most common material, perhaps, of which these fighting boomerangs are made is gidyea (*Acacia homalophylla*, A. Cunn.), though other woods, such as mulga (*Acacia aneura*, F. v. M.), white-gum, &c., are often used, the name of the timber occasionally giving the name to the implement [...]. The weapon is usually cut out from the side of a tree-trunk *en bloc*, then gradually got into shape with a chisel, &c., and finally smoothed off with a piece of sharp-edged flint or glass. With wood of suitable grain, white-gum, for example, the original block may be split down and two boomerangs made of it. Any defect in shape, in the nature of a bend or twist, can be remedied by the various artificial means which have already been discussed (sec. 151). The mode of manufacture of the hooked variety varies somewhat from the preceding, the portion of the trunk for its shaft being cut out at the same time with an adjacent branch or rootlet for its hook ...

Roth [1909, Pl. LIX] indicates in a sketch the "method of cutting a boomerang from a flange on the butt of a tree". Smyth [1878, p. 311], referring to the returning boomerang (wonguim) of Victoria, states:

The woods commonly used for making boomerangs are the limbs of the iron-bark and she-oak, but the roots of the various kinds of eucalypti are in some places highly esteemed.
Very good boomerangs, of the class to which the *Wonguim* belongs, are sometimes made of the bark of the gum-trees. The bark is cut into the right shape, and heated in ashes and twisted slightly. Weapons made of bark may have a good flight, but they are not so valuable as those made of hard wood. Even those made of wood are not seldom heated, softened, and twisted; but the best *Wonguims* are cut with a tool into the right shape. The eye of the maker guides every stroke, and when the instrument is finished it is not necessary to heat it and bend it.

According to Nicols [1877, p. 510]:

The weapon is made of various woods, a piece with a slight elbow being selected. It is hardened by baking. The right form is arrived at by trial, as I have seen during the process of manufacture. Those sold to Europeans are the failures.

Campbell's [1958] documentary does not show any trials or flight tests being made during the manufacturing, but that such trials may be important is indicated by the following three quotations:

In form, in length, and in weight, the boomerangs which return vary a good deal. The men who are most skilful in shaping these instruments rarely make two of the same pattern. They are chipped and smoothed as experiments made from time to time suggest alterations, and the weapon is not finally completed until it has been thrown successfully, and has come back in the manner desired by the maker. [Smyth, 1878, p. 311]

The boomerangs are manufactured from greenwood cut to the desired shape and angle. The points are hardened by drying in hot sand or ashes, after which the weapon is bent to the required twist whilst held firmly on the ground by the ball of the foot and wrenched with the hand. But even after this treatment the boomerang is not finished until repeated trials of its flight have been made, and it is chipped, scraped, and twisted until its working qualities are considered perfect. [Jennings & Hardy, 1899, p. 627]

In the manufacture of the boomerang, the most expert could never tell whether or not the one he was making would be successful or not, until it was carved in the rough and he frequently had to throw away one after another before he succeeded in developing the necessary curve; when this is obtained he continued working at it until nearly finished, when he tested it in the open. If the flight was unsatisfactory he heated the faulty part in the hot ashes to make it pliable; he then held it between his teeth while his hands gave it the necessary twist. This he continued until he was quite satisfied. The only tools used were a stone tomahawk and pieces of quartz. [Christison & Edge-Partington, 1903, p. 38]

The last two quotations particularly stress the "required" or "necessary" twist (see §4), and it is indeed true that even apparently small differences in twist may cause large differences in the flying behaviour of boomerangs. The methods for producing the right amount of twist are also mentioned by Lumholtz [1889, p. 49/50]:

The peculiarity of the boomerang, viz. that it returns of itself to the thrower, depends on the fact that it is twisted so that the ends are bent in opposite directions; the twisting is accomplished by putting it in water, then heating it in ashes, and finally bending it, but this warp must occasionally be renewed, for it sometimes disappears, especially if the weapon is made of light wood.

I want to emphasize that this bending and twisting of boomerangs by Aborigines does not necessarily imply that a twist is being *produced*; it seems more probable that the extant twist is being *modified*:

The Australians in the manufacture of all their weapons, follow the natural grain of the wood ... [Lane Fox, 1868, p. 423]

It would be fortuitous if the grain of the wood from which a boomerang is made would be entirely without some twist of its own, or if this twist would be exactly of the nature and the amount desired for a good boomerang.

Before flight trials have been made an Aboriginal boomerang maker may not even know whether his boomerang will be a returning one or not. Howitt [1876, p. 248] remarks:

... I have great doubt whether any of the natives can tell beforehand whether a boomerang No. 1 [return type] will, when finished, be a good "marndwullun wunkun" [returning boomerang] or not; and it is not uncomcom for an aborigine, if he finds his boomerang to return instead of going straight to its mark, to heat it in the ashes and straighten it, so that the blades lie in one plane.

Mathew [1887, p. 158] states:

Of boomerangs they had two kinds; one returned when thrown, the other did not. Whether the weapon would return or not seemed the effect of chance, and not design, in the construction, for, without a trial, they seemed unable to tell which kind the weapon was, and the end to be held was indicated by a few scratches at the very extremity.

And Roth [1902a, p. 512] = [1902b, p. 19], writing about toy boomerangs in Queensland, says:

The boomerang is said to be right or left handed, not because it is necessarily thrown with the one or the other hand, but because it circles to the right or left side of the thrower. Until its manufacture is completed, and it has been tried by experience, the blacks have told me that they cannot determine with any degree of assurance which of the two varieties it will prove to be.

Broken boomerangs are sometimes repaired. According to Nevermann [!925, p. 43]:

Die Anfertigung eines Bumerangs kostet viel Zeit und Mühe. Deshalb werden zerbrochene Wurf- und Schlaghölzer sorgfältig ausgebessert, wenn es sich noch loht. Die Ausbesserung ist auf zweierlei Weise möglich. Entweder klebt man die einzelnen Stücke mit Harz zusammen, das aus Stachelschweingras gewonnen wird, oder man bindet sie mit nassen Känguruhsehnen aneinander. Die letztere Art soll die bessere sein.

A picture of a repaired boomerang is shown in [Cranstone, 1973, p. 16].

Do Aboriginal boomerang makers possess any knowledge concerning "the aerodynamic and mechanical principles" which determine the flying behaviour of boomerangs? The quotation marks are added here because the concepts used by Aborigines when they deal with physical phenomena may be quite different from those used at present by scientists. In the available literature I did not find any information concerning this question, except for a rather vague indication by Nevermann [1925, p. 40], relating to the Aranda in Central Australia:

Eine genauere Erklärung für die sonderbare Flugwirkung des Kögarögana [returning boomerang] haben die Aranda selbst nicht. Sie halten seine Eigenart, die sie mbaritjika nennen, für eine Erfindung ihres Gottvorfahren, denn er zeigte dem ersten Menschen, wie man das Geheimnis der Wiederkehr in den Bumerang hineinzaubert.

Boomerangs could very well be manufactured according to certain empirical rules and routines only (more or less as bicycles are manufactured in our society, where, I would guess, most bicycle makers would not be familiar with the principles by which a bicyclist maintains his balance). On the other hand, Aboriginal boomerang makers might have a set of "theoretical" concepts which they apply in the manufacturing of boomerangs.

§6 *The throwing and the flight of Australian boomerangs.*

This section deals mainly with the throwing and the flight of *returning* boomerangs. The manner in which an Aborigine proceeds when he is about to throw a boomerang is nicely described by Smyth [1878, p. 312]:

> When a skilful thrower takes hold of a boomerang with the intention of throwing it, he examines it carefully (even if it be his own weapon, and if it be a strange weapon still more carefully), and, holding it in his hand, almost as a reaper would hold a sickle, he moves about slowly, examining all objects in the distance, heedfully noticing the direction of the wind as indicated by the moving of the leaves of trees and the waving of the grass, and not until he has got into the right position does he shake the weapon loosely, so as to feel that the muscles of his wrist are under command. More than once as he lightly grasps the weapon he makes the effort to throw it. At the last moment, when he feels that he can strike the wind at the right angle, all his force is thrown into the effort: the missile leaves his hand in a direction nearly perpendicular to the surface; but the right impulse has been given, and it quickly turns its flat surface towards the earth, gyrates on its axis, makes a wide sweep, and returns with a fluttering motion to his feet. This he repeats time after time, and with ease and certainty. When well thrown, the furthest point of the curve described is usually distant one hundred or one hundred and fifty yards [90-140 m.] from the thrower.

It seems improbable that "in a direction nearly perpendicular to the surface" would refer to the boomerang's flight path; rather Smyth would mean that the boomerang's plane of rotation is nearly vertical at the start. Baker [1890, p. 377] provides the following lively description of boomerang throwing by an Aborigine near Botany Bay:

> Arrived at a safe distance from the camp, he braced himself, and saying, "Look out, boss!" ran forward two or three steps, bent his body backward in the form of a bow, brought the boomerang over between head and shoulder, then hurled it into space. The moment it left his hand it looked like a wheel revolving rapidly in the air, and made a harsh, whirring sound. Taking a circle about one hundred and fifty yards [140 m.] in diameter, it passed around to the left, turning backward in a gradual curve, and struck the ground a few yards from us, sending up a cloud of sand.

The distances reached by returning boomerangs as mentioned by both Smyth and Baker (and many other authors as well) are probably seriously overestimated. This point will be discussed further down in this section.

Another description by Smyth [1878, p. 312] stresses the surprising turns in the flight of a returning boomerang:

I have seen the natives at Coranderrk throwing the *Wonguim* on many occasions; and the skilful thrower seemed to be able to do exactly what he liked with the weapon. He would throw a thin blade in such a way as to make it almost disappear in the distance - indeed, when the edge was presented, it was for a moment or two impossible to follow the flight with the eye - it would then return, gyrate above the thrower in an absurd manner, descend and describe a curve as if it were about to strike him, go off in another direction, still descending, so as to alarm a group of blacks at a distance, and fall finally some yards behind him; the thrower, the while, regarding the weapon with an intelligent and amused expression, as if he knew exactly where it was going and where it would fall.

Sofar these quotations provide only a general impression of the throwing and the flight of returning boomerangs. A more precise description of the throwing technique is given by McCarthy [1957, p. 88]:

It is thrown with a vigorous action in which the thrower runs a few steps to gain greater impetus. The weapon is held at one end, behind the head, with the convex surface to the left and the concave edge to the front, swung rapidly forward, and just before release is given additional impetus by a powerful wrist movement.

This wrist movement needs not be an active or "powerful" one; a sudden stop of the throwing hand just before releasing the boomerang will do [Hess, 1968a, p. 126]. It is a curious fact that Aboriginal boomerang

fig. 6.1 Throwing a right-handed returning boomerang. (Copied from [Ray, 1906, p. 87].) From left to right: thrower's arm brought behind the shoulder; moment of release; afterwards.

throwers invariably hold the boomerang in such a way that the free extremity points forward at the moment of release, i.e. "with the concave edge to the front", see fig. 6.1. The boomerang is thus held at the end denoted by 1 in fig. 4.1. But why could not a boomerang be thrown with the convex edge in front? The choice would be only a matter of personal preference, it seems to me [Hess, 1968a, p. 126]; and according to Hawes [no date]:

It doesn't make much difference which end you hold as long as the flat side is away from you and the boomerang is inclined at the correct angle and thrown with spin! It can't remember which end you held after it has left your hand!!

Perhaps one end of an Australian boomerang (end 1 in fig. 4.1) generally would be better suited to be used as a grip than the other end:

When about to throw, he grasped the boomerang firmly in his right hand, holding it by its extremity, which, as is the case with every come-back boomerang, was slightly roughened to afford a firm grip, with the flattened surface towards the palm. [Jennings & Hardy, 1899, p. 627]

But it is difficult to see why only end 1 should be roughened this way. Since boomerangs may be used for a great variety of purposes (see §7), the reason behind the uniform Aboriginal way of throwing might be found in the use of the boomerang not as a throwing implement, but as some other tool. It might be worthwhile to investigate this matter in the field in places where boomerangs are still thrown by Australian Aborigines.

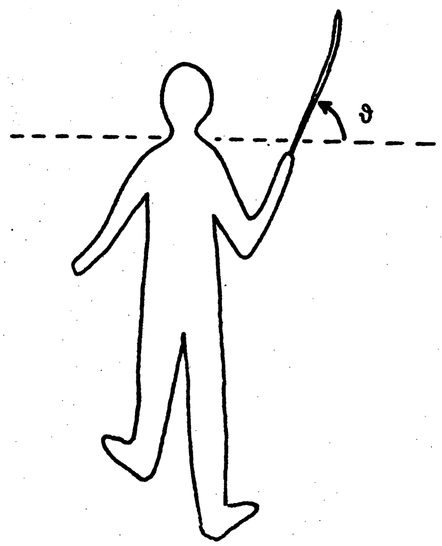


fig. 6.2 Right-handed boomerang thrower; view from behind. ϑ is the angle between the boomerang's plane of rotation and the horizon.

The angle between the boomerang's plane of rotation and the horizon at launch (ϑ in fig. 6.2) has a profound influence on the boomerang's flight path [Hess, 1968a]. Most references to Aboriginal boomerang throwing contain only scanty information on this angle. Probably the boomerang's plane should be vertical or inclined with the upperpart to the right of the right-handed thrower ($45^\circ \lesssim \vartheta \lesssim 90^\circ$) in order to obtain good flights with most returning boomerangs. However, Howitt [1876, p. 249] mentions:

I found that the throws could be placed in two classes, one in which the boomerang was held when thrown in a plane perpendicular to the horizon, the other in which one plane of the boomerang was inclined to the left of the thrower.

And the same author [1877, p. 313] quotes a mr. James as follows:

In throwing the boomerang I have seen it usually held nearly parallel with the horizon. When thus thrown it would rise and return towards the thrower ...

Straight-flying fighting and hunting boomerangs probably are thrown with their plane of rotation approximately horizontal. Some, possibly secondhand, information on this point is given by Sarg [1911, p. 16]:

Im Gegensatz zum Spiel-Bumerang wird die zur Jagd und Krieg dienende Waffe beim Wurfe horizontal gehalten und unter dem in der Linken gehaltenen Schild hervorgeschleudert; auch sie dreht sich während des Fluges um ihre Querachse, ihre Rotationsebene aber bleibt, da sie keinen Drall hat, horizontal mit nur leichter Neigung zum Aufwärtssteigen, die Flugbahn hält daher die beim Loslassen erteilte Richtung inne, bis die Triebkraft erschöpft ist.

It is regrettable that so very few descriptions of the throwing and the flight of Aboriginal boomerangs are accurate and unambiguous. If a direction or an orientation is mentioned it is not always clear whether the direction of the boomerang's forward motion is meant or the orientation of its plane of rotation. (Consider for example the quotation from [Smyth, 1878] at the beginning of this section.) It is difficult not to believe that Howitt [1876, p. 249] made a mistake when he described a method of throwing the boomerang with its plane "inclined to the left of the thrower". The more so as in his subsequent description of a boomerang flight the boomerang's plane, initially vertical, is said to become "inclined to the left", whereas most returning boomerangs would become inclined to the right: the angle ϑ would decrease. See among others: [Smyth, 1878, p. 312], [Thomas, 1906, p. 78; 1955, p. 883], [Hess, 1968a, p. 128/30]. But here Howitt could have referred to another, quite essential, change of orientation: the boomerang's plane of rotation turns with its foremost part to the left: counterclockwise as viewed from above (see for instance [Hess, 1968a].) The complete description by Howitt [1876, p. 249] reads as follows:

In the first method of throwing [boomerang's plane vertical], the missile proceeded, revolving with great velocity, in a perpendicular plane for say 100 yards [90 m.], when it became inclined to the left, travelling from right to left. It then circled upwards, the plane in which it revolved indicating a cone, the apex of which would lie some distance in front of the thrower. When the boomerang in travelling passed round to a point above and somewhat to the right of the thrower, and perhaps 100 feet [30 m.] above the ground, it appeared to become stationary for a moment; I can only use the term *hovering* to describe it. It then commenced to descend, still revolving in the same direction, but the curve followed was reversed, the boomerang travelling from left to right, and the speed rapidly increasing, it flew far to the rear. At high speed a sharp whistling noise could be heard. In the second method [boomerang's plane inclined to the left], which was shown by "bungil wunkun," [= "He of the Boomerang", the thrower's nickname] and elicited admiring ejaculations of "ko-*ki*" from the black fellows, the boomerang was thrown in a plane considerably inclined to the left. It there flew forward for say the same distance as before, gradually curving upwards, when it seemed to "soar" up - this is the best term - just as a bird may be seen to circle upwards with extended wings. The boomerang of course was all this time revolving rapidly. It is difficult to estimate the height to which it soared, making, I think, two gyrations; but judging from the height of neighbouring trees on the river bank, which it surmounted, it may have reached 150 feet [45 m.]. It then soared round and round in a decreasing spiral and fell about 100 yards [90 m.] in front of the thrower. This was performed several times. The descending curve passed the thrower, I think, three times. Other throws were spoiled by the wind, which carried the boomerang far to the front. I observed, and some of the aborigines confirmed it, that the thrower preferred throwing with the wind.

The quoted passage seems to be a qualitative report and the distances and heights mentioned in it are very rough estimates only, as Howitt himself makes clear. Nevertheless it has apparently served as a basis for descriptions by others: Walker [1901a, p. 457] = [1901b, p. 338] = [1901c, p. 515], Thomas [1906, p. 78], [1910, p. 236] = [1955, p. 883], Cornish [1956, p. 245], McCarthy [1958a, p. 44]. For example, consider this description by Thomas [1955, p. 883]:

Throws of 100 yds. [90 m.] or more, before the leftward curve begins, can be accomplished by Australian natives, the weapon rising as much as 150 ft. [45 m.] in the air and circling five times before returning.

Although it is not impossible for some returning boomerangs to reach distances of 90 m. or more (see §13), a typical return flight path would have a maximum diameter of about 30 m. and a maximum height of some 15 m., roughly speaking. To make a boomerang attain a height of 45 m. would be difficult to accomplish, even for an expert thrower. As I remarked earlier in this section, the dimensions of boomerang flight paths often are seriously overestimated. Distances of over 100 m. reached by returning boomerangs are not seldom reported. In none of these cases is there any indication that such distances were actually measured. Probably rough estimates were made by untrained eyewitnesses; and it is surprisingly easy to overestimate the distance of a fast flying and spinning piece of wood high up in the air. I could find only one second-hand reference to measurements of flight paths traversed by Australian Aboriginal boomerangs; Thomas [1906, p. 79] states:

> The distance to which the return form can be thrown is a matter of much dispute. Howitt describes a throw of one hundred yards [90 m.] (estimated), but there can be no doubt that this has been exceeded. I have been informed by a resident at Coranderrk, that he has measured throws of one hundred and twenty yards [110 m.], and seen throws of over one hundred and fifty yards [140 m.]. The war boomerang can probably be thrown two hundred and fifty yards [230 m.] or more.

Unfortunately, the method of measuring is not indicated, and neither is mentioned whether or not the boomerang(s) fully returned to the thrower.

It would not be a difficult matter to make a reliable estimate of the dimensions of a boomerang's trajectory, by placing one witness at the point over which the boomerang is expected to reach its farthest distance. The right position can be found by trial and error after a few throws. Afterwards the distance between witness and thrower can be determined with a tape measure. By means of a protractor the angular elevation of the boomerang can be determined by a second witness. The combination of both measurements would provide a reliable estimate of the boomerang's height. It is astonishing that even such simple procedures are never mentioned in the ethnographical literature, and apparently have never been carried out as far as Aboriginal boomerang throwing is concerned.

Representations of flight paths in ethnographical publications are scarce and mostly inaccurate. Some sketches are provided by Roth [1897, Pl. XVII], [1902, Pl. XXXII], Jennings and Hardy [1899, p. 627/9] and Ray [1906, p. 88/9]. Some of the flight paths depicted in these last two references are quite fanciful. The sketches by Baker [1890, p.376/7] appear to be reasonable, qualitative representations, but are based on Baker's throwing his own boomerangs himself. Outside the ethnographical literature far more accurate representations of flight paths, traversed by self-made boomerangs, have been published by Erdmann [1869], Walker [1897, 1900, 1901a,b,c], Buchner [1905, 1916, 1918] and Hess [1968a]. However, this latter group of publications does not provide direct information on the flight of *Aboriginal* boomerangs.

Comparative flight tests with Aboriginal boomerangs from different regions apparently have never been carried out. Only Smyth [1878, p. 336] mentions:

All the West Australian boomerangs seem to fly further than those used by the natives of the east.

Returning boomerangs sometimes are thrown in such a way that they hit the ground:

It can be thrown so as to run along the ground for some distance, hoop-fashion, then ascend, describe a great curve, and return to the thrower. [Smyth, 1878. p. 312]

A boomerang should be strong enough for this; Howitt [1876, p. 249] relates:

Another method of throwing was mentioned, namely, to throw the boomerang in such a manner that it would strike the ground with its flat side some distance in front of the thrower. It would then rise upwards in a spiral, returning in the same. This was not attempted as it was decided the boomerang was not strong enough. A final throw in a vertical plane so that the missile struck the ground violently fifty or sixty yards in advance terminated the display. It ricocheted three times with a twanging noise and split along the centre.

As was mentioned in §2 and §4, boomerangs to be thrown with the left hand, i.e. with opposite spin, also exist; according to Smyth [1878, p. 311]:

The *Wonguim* returns to the feet of the thrower when skilfully thrown.

Generally it is so fashioned as to describe a curve from right to left; but one in my possession, which I have seen thrown with precision, so as to return every time to within a short distance of the thrower, is a left-hand boomerang. It describes a curve from left to right.

A left-handed boomerang can be considered in every respect, including its flight path, as the mirror image of a right-handed boomerang [Hess, 1968a, p. 127/8]. As to the occurrence of left-handedness among the Australian Aborigines (in North West Central Queensland) Roth [1897, p.143] remarks:

Among the number of aboriginals examined for left-handedness, the proportion of such was found to be very marked ...

The distinction between the flights of returning and fighting boomerangs may not be very sharp; Roth [1897, p. 129] points out that

... any kind of fighting boomerang can be more or less thrown in such manner as to "return" or "come-back," making one single complete, or perhaps two incomplete, revolutions from the starting-point.

The influence of the prevailing wind on the flight of returning boomerangs is described by Charnay [1878, p. 72] as follows:

D'abord, le boumerang ne revient bien qu'avec le vent, dont l'Australien étudie avec soin la direction; car, s'il ne jette pas son arme exactement dans le vent, elle ne reviendra pas sur lui, mais ira tomber à gauche et à droite, selon que le boumerang est gaucher ou droitier, ce qui tient à une légère inflexion du bout de l'arme. Si le vent est trop fort, le boumerang volera très loin en arrière et ne reviendra pas au point de départ; enfin, s'il n'y a pas de vent du tout, le boumerang, après avoir épuisé sa force de projection, s'enlèvera en tournoyant sur lui-même et décrira des cercles presque concentriques, en descendant au fur et à mesure que s'éteindra sa force de rotation.

To conclude with, here follow some remarks concerning the spectators rather then the throwers:

It is dangerous to stand near the thrower, if the observer have not selfpossession. When the instrument returns, it is necessary to look at it attentively, and not to move unless it comes too nigh; any hurried movement, due to alarm, for the purpose of avoiding it, might result in its striking the affrighted person and inflicting a serious wound. The plan is to stand quite still, and to wait patiently until the force is expended. The thrower, if skilful, will take care that, if the observers keep their places, none of them are injured. [Smyth, 1878, p. 313]

In 1885 some Australian Aborigines were "showed" in Münster, Germany by an American impressario. The show included a demonstration of boomerang throwing, in which a curious incident took place:

> Nur ein einziges Mal machte ein Bumerang mit dem Hute eines Zuschauers unbeliebsame Bekanntschaft, indem er auf dem Rückwege zur Erde denselben wie mit einem Rasirmesser haarscharf mitten durchschnitt, so dass die obere Hälfte des Hutes vom Kopfe flog; auch die Brille des betreffenden Herrn ging in Stücke. [Landois, 1885, p. 544]

§7 *Various uses of Australian boomerangs.*

This section deals with the different ways in which boomerangs are used by the Australian Aborigines. As has already been made clear, returning boomerangs are mainly used as toys. For instance, Smyth [1878, p. 311], referring to the returning wonguim of Victoria, says:

The boomerang here decribed is usually regarded as a plaything: it is not a war-boomerang; and though it is occasionally used in battle, and sometimes for killing birds and small animals, it is not so handy as the short stick named *Konnung*.

And according to McCarthy [1958a, p. 44]:

The returning boomerang is used chiefly as a toy in tournaments, the object being to see which man can make it accomplish the greatest number of circles in flight, and bring it back nearest to the thrower or to a peg set in the ground.

The most detailed account of the returning boomerang being used for play is provided by Roth [1897, p. 128] (North West Central Queensland):

The return boomerang is never used as a recognised article of exchange or barter: that is to say it does not travel, and is manufactured just as occasion requires. It is strictly a man's toy, and is used in different ways as follows: - In the Boulia District, five, six, or perhaps more men will stand in Indian file, each individual with raised arms resting his hands on the shoulders of the one in front: another of the playmates, standing by himself at some distance ahead and facing the foremost of the file, throws the boomerang over their heads, and as it circles round they all follow it in its gyration, the game being for any of them to escape being hit, each taking it in turn to throw the missile [...]. Among the Yaroinga tribe on the Upper Georgina, they often try and arrange to make up two sides, the object being for a member of one team to hit an individual of the opposite group. In the Cloncurry District, the Mitakoodi fix a peg into the ground, and the one who can strike or come nearest to it with the boomerang when it falls to the ground is declared the best man.

However, boomerangs of the returning type may occasionally be used for hunting. Howitt [1876, p. 248] states:

In Cooper's Creek I have seen boomerangs No. 1 [return type] used by the natives to kill ducks and birds in general which fly in flocks. They seemed unable to calculate where its course would be among them, and some were hit; the boomerang and the bird both fell.

And Oldfield [1865, p. 264/5] writes:

Where such birds congregate largely, the boomerang is of essential use; for a great number of them being simultaneously hurled into a large flock of water-fowl, ensures the capture of considerable numbers, but, unless under such circumstances, its value is inconsiderable. No native ever attempts to kill a solitary bird or beast by means of the boomerang, for even in the hands of those most expert in its use, its effects are very uncertain.

A somewhat more subtle and indirect use of the returning boomerang as a means of catching fowl is mentioned by McCarthy [1958a, p. 44]:

It is also thrown in the air to imitate a hawk flying over a flock of ducks, cockatoos or parrakeets, so causing the birds to dip downward and to be caught in a net spread either across a creek or a break in the forest.

A detailed and lively description of this method is related by Waite [1930, p. 436/7]. See also [Krefft, 1866, p. 368/9]. The kaili or Western Australian boomerang, which is not always of the returning kind, is used for killing birds as well as fish:

Kockatoos are killed by throwing the *kaili* [...], among a flock when flying in dense clouds along the creek or sitting in thick clusters on the branches of the gum-trees; and they rarely miss. [Clement, 1904, p. 3]

In shallow water, the fish are killed by throwing the *"kaili"* [...] at them, when they are 5 or 6 inches below the surface. The writer saw a native in Cossack-creek kill eleven fishes, weighing from one to two pounds each, in less than half an hour, by means of the *kaili*. [Clement, 1904, p. 3/4]

See also [Peggs, 1903].

Basedow [1925, p. 168] describes the use of boomerangs as striking weapons in duels:

Where the boomerang is known it, too, is extensively used, in conjunction with the shield, by duellists to settle minor altercations. The offended party throws one of his missiles into the camp of his rival as a summons to the fight, whereupon the latter immediately responds by throwing another back, and walks out into the open, carrying with him a single boomerang and a shield. Both men now start a war-dance, during which they gradually approach each other, lifting their legs high in the knees, brandishing their boomerangs in the air, and holding their shields in front of their bodies. After a while, they close in; and the real fight begins. Whenever an uncovered spot presents itself on either man, the opponent, with the quickness of lightning, attempts to strike it with his weapon. The hands in particular are selected as the best marks to quickly put the rival out of action; and this opportunity is never missed when it presents itself to the quick eye of the native.

In revenge parties, boomerangs may be used in a more spectacular way:

In districts where the boomerang is used, a number of these weapons is carried in the belts of the belligerents. When the parties are within seeing distance of each other, each side begins to throw its boomerangs, making them fly high in the air towards the enemy and return to their respective owner. The demonstration is repeated time after time, as the contending parties draw near to each other, until at length the boomerangs fly well over the opponent's heads on either side. This is forsooth an awe-inspiring spectacle and has the desired effect of arousing the fighters' ire to a very high pitch. At a later stage, boomerangs are employed in actual battle. [Basedow, 1925, p. 187/8]

McCarthy (personal communication, 1974) remarks as to the importance of the boomerang as a hand weapon:

In the rock paintings and engravings men are more commonly armed with the boomerangs (one and a shield, one in a girdle, just one boomerang or up to three in each hand, etc.) than with spears and clubs. In eastern Australia, where the emu-kangaroo cord hunting nets were in use, the boomerang as a killing weapon appears to have supplanted the spear.

Apart from being used as hunting and fighting weapons or playthings, boomerangs may serve quite a variety of different purposes. According to McCarthy [1961, p. 348, with illustrations on p. 347]:

The fluted central Australian boomerang has a wider range of uses than any other type. It has one sharp end with which the men cut-open animals and chop them up, dig wells, fire-pits for cooking kangaroos and emus, and holes to uncover totem stones, unearth honey ants, lizards and other burrowing animals, and scrape the hot ashes over and away from cooking carcases. The edge of the boomerang is used as a fire-saw on a softwood shield, as a fabricator to retouch stone adzes, and, as a bow, is rubbed across the edge of another boomerang to produce a curious musical sound. This boomerang thus takes the place of several other artifacts that would have to be made and carried, and it illustrates an important principle in the life of the desert or spinifex country tribes of the interior. To them a reduction in the chattels to be carried means less weight to transport and more freedom of movement, vital needs for a semi-nomadic people who have to travel great distances between waterholes and in the search of food in a harsh environment. For this reason, the use of this versatile boomerang spread from the central Australian and Northern Territory tribes to those in western Queensland, Western Australia and South Australia, and until the white man destroyed tribal life it was rapidly replacing other kinds of boomerangs in these areas.

All of these uses of the fluted boomerang are demonstrated in Campbell's [1958] 16 mm documentary. Boomerangs throughout Australia are often used as musical instruments (clapping sticks):

The flatter faces of two are gently rapped together to mark the rhythm of a song or dance all over Australia. [McCarthy, 1961, p. 348]

Elkin [1953, p. 2], referring to Arnhem Land, says:

Indeed, boomerangs have been imported into this region as musical percussion instruments.

Van der Leeden [1967, p. 15] gives a rather detailed description of this use by the Nunggubuju in eastern Arnhem Land:

De platte, brede boemerangs produceren een veel scherper geluid dan de rechte, dikke clapping sticks, die elkaar bovendien slechts op één plaats raken. De boemerangs houdt men zodanig vast - het gebruik van boemerangs als clapping sticks verklaart overigens waarom men er liefst twee tegelijk van maakt - dat deze elkaar op twee plaatsen, aan beide uiteinden, raken. Dit geschiedt nooit precies op hetzelfde moment, en daardoor heeft het ritmische spel van de boemerangs ook een snel en jagend karakter.
[Translation: The flat, broad boomerangs produce a much sharper sound than the straight, thick clapping sticks, which, moreover, touch each other in one place only. The boomerangs are held in such a way - the use of boomerangs as clapping sticks explains why they are preferably manufactured in pairs - that they touch each other in two places, at both ends. This never happens at exactly the same instant, and because of this the rhythmic play of the boomerangs has a fast and driving character.]

See also [Basedow, 1925, p. 374, 383, pl. LII]. The remarkable fact that boomerangs are often manufactured, kept and traded in pairs has also been noticed by Roth [1897, p. 143]:

It is interesting to note that both in camp or on the walk-about, though an aboriginal may carry one spear, one shield, &c., he almost invariably has two boomerangs. If they have both been made by the same person they are very probably similarly marked: if he barters them, he will generally "swap" them only as a pair, though beyond the fact of two being required as an accompaniment for beating time at the sing-songs and the corrobborees, it is difficult to understand why this should so often be the case.

Different types of boomerangs can be ceremonially used in rituals. For example:

It is this boomerang [jilparindji or kalawal] [...] which is used in ritual defloration on the Kunapipi ground, before ceremonial coitus. That is to say, young girls have their hymens pierced prior to intercourse on the ceremonial ground; other, whose hymens have already been broken through normal premarital coitus, have the end of the boomerang placed symbolically in the vagina. [Berndt, 1951, p. 67]

According to McConnel [1935, p. 49/50] ceremonial cross-boomerangs are used in a "boomerang-dance" by the Kúng'gà:ndyi near Cairns:

The painted cross-boomerangs (yíntyo:r gídyar) are used in a dance. A stick fastened into the back of the yíntyo:r is twirled between the two hands to make the yíntyo:r revolve as does the boomerang when spinning through the air. This is done to a rhythmic movement as the men dance in a circle to a song. This spinning movement in the dance has earned for the yíntyo:r the popular name of "windmill" or "aeroplane." Needless to say these latter objects were unknown to the Kúng'gá:ndyi when this dance originated. The movement depicts the spinning of the cross-boomerang in the air, as when thrown in play at night with a live coal attached, for the amusement of the children.

§8 *The present situation in Australia.*

In the preceding sections often the present tense was used, whereas the past tense would have been more appropriate: the Aborigines' culture has vanished to a great extent. According to McCarthy [1961, p. 343]:

> There are not many places in Australia now where boomerangs are still made and used by Aborigines who follow their old way of life. These places are in the most remote parts of the continent where white contact has not disrupted the Aborigines' culture. Boomerangs may still be obtained in the Northern Territory, the desert areas of Western Australia, parts of the Kimberleys and adjoining areas, and in the southern portion of Cape York. Most of these are non-returning boomerangs, the returning type having almost completely disappeared except in places like Palm Island, La Perouse and elsewhere where it is made for the souvenir trade.

As regards the possibilities of studying Aboriginal boomerangs in the field Mr. F.D. McCarthy (personal communication, 1973) remarks:

> Comparative tests in several localities are required and time is running out rapidly for such research to be done. It is however still possible to work with Aborigines in the Northern Territory at places like Papunya, Yuendumu, Hooker Creek and other settlements who can make and throw non-returning boomerangs in the old style, and at Yalata, South Australia, and Warburton, Jigalong, Sunday Island and other places in Western Australia, where returning boomerangs were used.

And:

> La Perouse in Sydney would be a good place to study the throwing of boomerangs made for the tourist trade, as would Cherbourg in south-east Queensland, and Cundeelee in Western Australia.

An example of the present situation in relation to the manufacturing of boomerangs is provided by Rose [1965, p. 42/3] in his study of the Aborigines at Angas Downs, Central Australia, in 1962:

> Non-returning boomerangs were previously made by these people and used as a hunting weapon, according to the enquiries made by the writer. There were none however in the camp that were made for this original purpose. The only boomerangs that were made were for the tourist trade. For a serviceable boomerang it is necessary to select a suitable bent tree or branch so that the boomerang can be so shaped that the curve follows the grain of the wood. It clearly requires some experience and trouble to find such a curved branch. But the demands of the tourist trade, where the boomerang is merely a commodity in a money economy, are quite different from those where the boomerang is a hunting weapon or a means of production. As a consequence the wood (mulga) for the trade boomerangs was not selected carefully and most boomerangs sold (there were a few exceptions) were cut from straight pieces of relatively thick

wood. In the finished article the grain did not follow the curve. One of the difficulties that the Aborigines had, when offering the trade boomerangs for sale, was that many of the tourists expected a returning boomerang, which the Aborigines at Angas Downs did not know how to produce. It was not unusual that the tourist might throw, or persuade the Aborigine to throw, the boomerang to prove its returning capabilities. Almost invariably, when the article hit the ground it broke, splitting along the grain.

Cultural changes are reflected in the making of boomerangs; according to Rose [1965, p. 77/8]:

Boomerangs were usually made by the men but women occasionally made them, but their efforts were extremely crude. [...]
Traditionally spears, woomeras, boomerangs and shields would have been made by men and if the women were to begin making them, they would have to overcome the old traditional attitude and probably equally important, would need to acquire the requisite skill to make them. In the case of the boomerang, the women had overcome the tradition, for they in fact made them in 1962 but had not acquired the skill, so that what they produced were almost caricatures of the boomerangs made by the men.

Nowadays boomerangs are commercially produced and sold in Australia, not only by Aborigines, but also by "European" Australians. As an example consider Lorin Hawes of Mudgeeraba, Queensland:

He employs seven workers, who turn out 60,000 boomerangs a year. Most are sold in gift shops in major Australian cities, but a quarter of the output is shipped to North America and Europe for sporting clubs and wives whose husbands have everything else. In addition, about 150,000 paying tourists a year turn up at Hawes' bushland farm, which he calls a "boomerangery". [...]
All this is too much for Hawes' biggest competitor, the Queensland Department of Aboriginal and Island Affairs, which every year sells 40,000 boomerangs made by aborigines living on missions. [Hawes, 1972, p. 53]

§9 *Boomerangs outside Australia (1).*

Objects resembling boomerangs have been found in various parts of the world, notably in ancient Egypt, south India, north Africa, prehistoric Europe and among the Pueblo Indians in Arizona. In many of the reported cases it is difficult, or even impossible, to assess the function for which the object under consideration was designed, or the way it was used. It is no exception that a so-called boomerang of non-Australian origin has none of the aerodynamic properties necessary for a boomerang-like flying behaviour, but only some superficial resemblance to Australian boomerangs. A straight stick with round cross section clearly is not a boomerang. A curved stick, or a flat object with a curved planform can be thrown so as to spin stably in its plane (see §17). This may serve two purposes: first, with a flattened section the air resistance is reduced throughout the flight; secondly, the target is hit by one of the leading edges of the rotating weapon, which may be sharpened. Such curved implements still need not be boomerangs. Only if an object has such a shape that, rotating in its flight, it is *carried* by the air like an airplane or a helicopter, it should be called a genuine (returning, non-returning or straight-flying) boomerang.

Let us consider one by one the regions outside Australia were "boomerangs" reportedly have been found.

1. Tasmania.

Almost straight hunting sticks with round cross sections were in use among the Tasmanians according to Noetling [1911], who describes these sticks, called lughrana, in minute detail. They have a length of some 60 cm. and a thickness of about 2.5 cm.

One of the most interesting observations as to the way the lughrana was thrown is that of Backhouse, who states that they threw it "with a rotatory motion." This is confirmed by Breton, who says: "It can be thrown with ease forty yards [36 m.], and in its progress through the air goes horizontally, describing the same kind of circular motion that the boomerang does, with the like whirring noise." [Noetling, 1911, p. 71]

Noetling considers the lughrana as a primitive form of boomerang, but it certainly does not at all possess a boomerang's characteristic properties.

Apparently boomerangs were unknown to the Tasmanian Aborigines. Yet one boomerang has been found on this island near East Devonport in 1851 [van Gooch, 1942]. From the description by van Gooch [1942, p. 22, pl. VII] it is evident that this object very much resembles, and even may be, an Australian boomerang.

2. New Zealand.

In 1925 "an undoubted boomerang" was found in a recently exposed kitchen-midden at Muriwai Beach, Auckland [Hamilton, 1926]. The boomerang resembles a common type found in Queensland, has a length of 47 cm., a width of 3.8 cm. and a thickness of 0.75 cm. The Muriwai boomerang weighs 70 g. "Whatever the wood is it is certainly not a hardwood such as *Eucalyptus*." [Hamilton, 1926, p. 45]. It is not known from where this object came.

3. New Hebrides.

Rivers [1915] reports the use of "boomerangs" on the northern part of the west coast of the island Espiritu Santo:

> They are used entirely in sport. They do not return to the thrower, but show the deflections from a straight course which are characteristic of the flight of the Australian boomerang. In one method of throwing, the instrument is made to strike the ground a few yards in front of the thrower. One of the highest and longest throws seen by us was of this kind. [Rivers, 1915, p. 107]

The ends of these non-returning boomerangs are truncated, not rounded or pointed as with Australian boomerangs (see the pictures in [Rivers, 1915, p. 107]). Rivers gives no information on the cross sections of these objects.

4. Celebes.

On this Indonesian island different types of boomerang-like objects are found. On the Macassar peninsula throwing sticks with a knee-shaped bend are used to kill or scare away birds. Von Hoëvell [1902, p. 201/2 relates:

> Op een mijner laatste inspectie-reizen in the Noorderdistricten van Zuid-Celebes zag ik, door den Controleur H.P. Wagner daarop opmerkzaam gemaakt, te Pangkadjene eenige opgeschoten knapen bezig op de sawah's met kromhouten naar vogels, zoowel op stilzittende als in de vlucht, te werpen. Met verwonderlijke juistheid wisten ze de dieren te treffen en ze de vleugels of de poten lam te gooien, zoodat ze dan gemakkelijk te vangen waren.

De kunst om aan die werphouten een richting te geven, dat deze terugkeerden naar de plaats van waar ze geworpen werden, zoals met den echten boomerang het geval is, verstonden ze echter niet.
[Translation: On one of my recent inspection travels in the Northern districts of South-Celebes I saw, after my attention was drawn to it by the Controleur H.P. Wagner, some strapping lads on the rice fields at Pangkadjene engaged in throwing knuckle-timbers at birds, still sitting as well as flying ones. With amazing precision they knew to hit the animals and to cripple their wings or legs, so that they were easy to catch then. The art to give the throw-sticks a direction so that they would return to the place from where they were thrown, as is the case with the true boomerang, they did not know.]

Pictures of these knee-shaped throwing-sticks are published by von Hoëvell [1902, p. 201], Sarasin & Sarasin [1905, p. 231] and Kaudern [1929, p. 136/8]. Only Kaudern provides drawings of cross sections of some of these implements, which appear to be mostly circular and sometimes lozenge-shaped. It is evident that such sections are not at all suitable for objects to be carried by the air over long distances. Hence these throw-sticks should not be classified as (non-returning) boomerangs.

A quite different kind of boomerang-like implement from Celebes is described by Kaudern [1929, p. 230] as follows:

During my stay at the villages of Kelei and Taripa in Ondae in E. Central Celebes I saw children playing with flat pieces of split bamboo, which they called *tela*. These *tela* measure 20 cm. by 2,5 cm., they have square ends, and the edges of the long sides are slightly rounded off. One side of the *tela* is convex, the other side slightly concave in the middle [...]. The player took two *tela* in his left-hand between his thumb and forefinger, possibly also using his middle finger, holding them like recumbent T. One *tela* should rest on the thumb, pointing toward the player, the other one on top of it at right angles as seen, in [fig. 9.1]. The player raised his left hand in a level with his face, or even higher, and with a smart lash of the bat the top *tela* was sent flying. A clever player knew how to make his *tela* revolve in the air so as to describe an almost elliptic trajectory and return to the place whence it started. At Kelei, where this sport, *motela*, was a popular amusement, there was a boy who was so clever that his *tela* always returned to him, so that he could hit it with his bat, but he was seldom able to make it return a second time.

As only one of the bamboo laths is launched, the "tela boomerang" would seem to be rather different from ordinary boomerangs: it is completely straight. Because of this it would be difficult to make it spin stably (see §17); the lath tends to rotate around its longitudinal axis. The function of the other lath held in the launcher's hand must be to provide

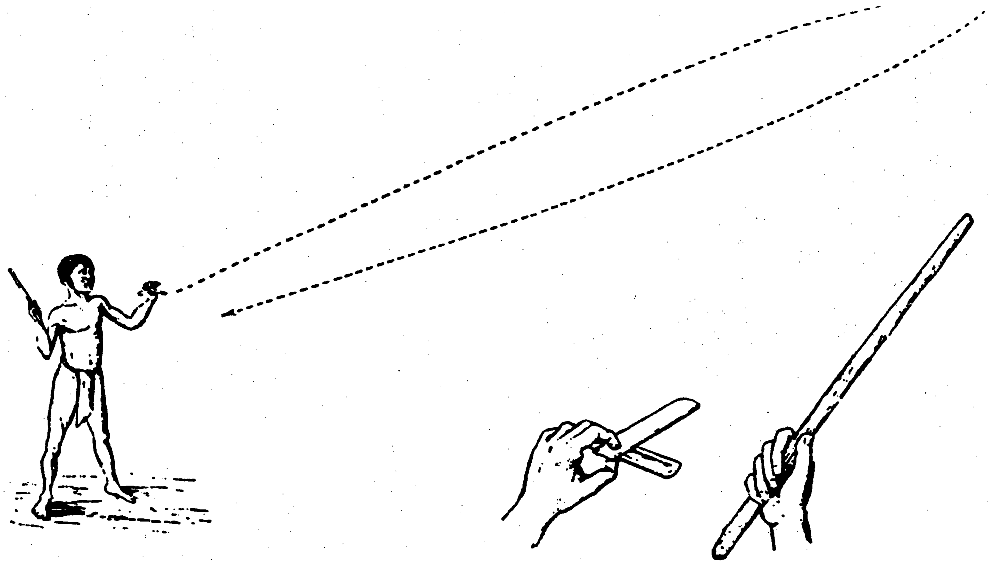


fig. 9.1 Boy playing the tela game.
(copied from [Kaudern, 1929, p. 232]).

a plane support at launch and thus prevent this wrong longitudinal rotation. In order to verify this point, I made a tela from plywood and tried to launch it in the manner described by Kaudern. With some skill it is indeed possible to make a tela spin like a boomerang, and probably (I did not fully succeed myself) to make it come back as well. After a bad start the tela soon spins rapidly around its longitudinal axis, and although this prevents the lath from returning, this rotation may generate enough lift to carry the tela in the air for a while. (A similar device is described by Mason [1937, Ch. V] under the name "tumblestick". The tumblestick can be thrown so as to come back, flying with an exclusively longitudinal spin. However, its mechanics are very much different from those of real boomerangs.) From Kaudern's description there can hardly be any doubt that the tela is a genuine boomerang-type implement, even though it is a border case.

A third type of boomerang-like implement found on Celebes is the cross boomerang, which was already mentioned in §2. Kaudern [1929, p. 235] describes it as follows:

At the village of Kelei in Ondae the boys also played with a kind of cross-shaped boomerang. This game they called *motela*, like the foregoing.
The cross is made of two flat splints of bamboo, closely similar to the above described *tela*. They are lashed together to make a right-angled cross with limbs of almost equal length. The limbs of one specimen in my collection, No. 2589, are 25 cm. by 3,3 cm. The corresponding measurements of a second specimen, No. 2590, are 27,5 cm. and 28,5 cm. by 3 and 3,2 cm. [fig. 2.3A]. The thrower with his right hand sends the cross into the air making it describe a curve approaching an ellips. Evidently it was far from easy to make a fine shot, at any rate it required greater skill than the game played with two *tela*. I only saw two boys who knew how to handle the cross boomerang properly.

As was mentioned in §2 and shown in fig. 2.3, if one limb has its convex side up during flight, the other limb has its concave side up. This would seem to be an aerodynamic disadvantage.

5. *America.*

The Hopi or Moqui Pueblo Indians in what is now called Arizona used a boomerang-like weapon known as "rabbit stick". Hough [1910, p. 348] gives the following description:

The flat, curved rabbit club, *pûtshkohu* of the Hopi, often called a boomerang, is not self-retrieving like the Australian weapon, though it shares the aeroplane nature of the latter; it is similar in form, but has not the delicate curves shaped to cause a return flight. [...] The Hopi rabbit stick is delivered in the same way as the Australian, and its course after it strikes the ground often brings it to the right or left of the thrower and nearer to him than the farthest point reached in its flight. It makes one or more revolutions in its flight toward a rabbit, and if it does not strike the animal directly, its rapid gyration when it touches the ground makes probably the hitting of any object within several feet. So far as is known this is the only aeroplane club used in America. The material is Gambell's oak (*Quercus gambelii*), and a branch of the proper curve is selected for its manufacture. One end is cut out to form a handle, and the club is usually varnished with resin and painted with a invariable design in black, red and green. [...].
The Gabrieleños of s. California used a rabbit stick similar to that of the Hopi; it was 2 ft [60 cm.] in length in a straight line, 1¼ in. [3.3 cm.] across at the handle, and 1¾ in. [4.6 cm.] across at the broadest part, with an average thickness of ¾ in. [1.8 cm.]. It was made of hard wood, and ornamented with markings burnt in the surface.

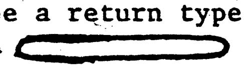
An earlier description of the rabbit stick and its use by the Moqui is given by Parry [1872, p. 399]:

It is a stick of hard wood flattened on both sides to a thickness of one-half inch [1.3 cm.], having an average width of two inches [5 cm.], and curved near the middle at an obtuse angle of 130°, furnished at one end with a handle. This was thrown by a direct whirling motion, aimed at the legs of the leaping rabbits, and generally, whenever the object was fairly presented, proving successful in bringing down the game by fracturing their legs.

Pictures of rabbit sticks are published by Stevenson [1880, fig. 548/9] and Cornish [1956, p. 244]. One reference, of doubtful value, to return behaviour of these implements is provided by Verrill [1927, p. 189]:

In its simplest form the boomerang or rabbit stick is merely a throwing club designed to knock over rabbits, gophers, and other small game. There is every gradation from these crude weapons to carefully curved and designed boomerangs which, if properly thrown, will travel in a circular path and return to the vicinity of the spot whence they started.

No details concerning the cross sections of these implements can be found in the literature, but from the measurements given by Hough follows a thickness over chordlength ratio of at least 0.42, whereas the measurements given by Parry indicate a value of 0.25 for this ratio. Anyway the rabbit sticks seem to be rather thick to be called "aeroplane clubs". Mr. G. Rayner, who recently examined some rabbit sticks in the San Diego Museum of Man, reaches about the same conclusion (personal communication, 1974):

Most cross sections were  or , spans about 2 feet [60 cm.] maximum, most about 18" [45 cm.], made of wood which the maker had bent to about 150 degrees included. I characterize them as only semi-aerodynamic, and saw nothing which even approached the craftsmanship and relative sophistication of the Australian aboriginal straight flying boomerangs in the museum's collection. Certainly the California rabbit sticks could not return, and the Hopi rabbit stick I saw, which was flatter, and appeared to be of recent construction, for ceremonial purposes, with its included angle of 150 to 160 degrees, and lack of twist, could not be a return type. The Hopi rabbit stick had a handle, and was of section 

Some of these implements might be carried by the air significantly farther than would have been possible with circular cross sections, and therefore some rabbit sticks might perhaps be considered as a simple kind of non-returning boomerang. Unfortunately, the available literature contains no information on the distance reached by these weapons. (See also [Callahan,1975].)

6. India.

Boomerang-like implements have been reported to be in use among Dravidian peoples in India. The following description by Hornell [1924, p. 336/8] seems to apply to most of the South Indian "boomerangs" (see fig. 10.1):

[They] have a gripping knob at the narrow or proximal end. Beyond this handle the width increases extremely gradually to the distal extremity, at the same time becoming laterally compressed to form a stout wide blade. The thickest part is usually a few inches from the distal end, which is abruptly truncate.

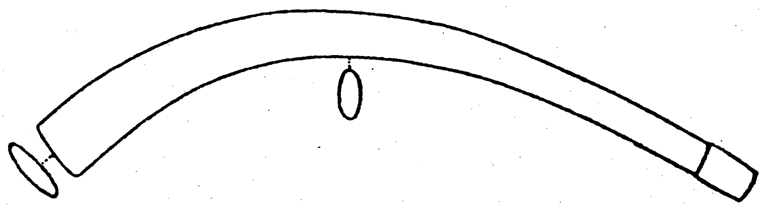


fig. 10.1 South Indian "boomerang", made of tamarind wood. (copied from [Hornell, 1924, p. 337])

Measurements and drawings of four South Indian "boomerangs", obtained in 1922 in the Ramnad district, are provided by Hornell [1924, p. 336/7]. Three of these have flattened (probably elliptical) cross sections. The tip-to-tip lengths are respectively 51, 51 and 75 cm., the widths at the distal ends 4.4, 4.2 amd 4.0 cm., and the thicknesses at the distal ends 1.7, 1.3 and 2.2 cm. They weigh some 500 g. With such thick cross sections these heavy objects could hardly be carried by the air. They might be throwing clubs rather than (non-returning) boomerangs or perhaps border cases.

As to the use of these implements Hornell [1924, p. 338] states:

The South Indian boomerang is employed primarily in hunting hares; deer and partridges are also sometimes struck down with this weapon. I was informed that the short, broad type is used for hunting all these animals, whereas the long one is never used for birds (partridges). None of these Indian boomerangs can be made to return to the thrower.

For ceremonial uses special "boomerangs" exist. Hornell [1924, p. 339], referring to specimens in the Madras Museum, says:

> Several of these are of ivory, one is of iron or steel [..], and another is of wood ornamented with metal work which would be at once damaged if used in hunting.

Apparently nothing is known about the range of these South Indian implements. By some it is claimed that they are capable of performing return flights. Thurston [1907, p. 559] quotes R. Bruce Foote, who found that these "boomerangs" were used in semi-religious hare hunting parties:

> Whether a dexterous Maravar thrower could make his weapon return to him I could not find out. Certainly in none of the throws observed by me was any tendency to a return perceptible. But for simple straight shots these boomerangs answer admirably.

But Oppert [1880a, p. 19] (also quoted by Thurston [1907, p. 556]) asserts, without adducing evidence:

> Their name in Tamil is *valai taḍi* [...] bent stick, as the stick is bent and flat. When thrown a whirling motion is imparted to the weapon which causes it to return to the place from which it was thrown. The natives are well acquainted with this peculiar fact.

And Lane Fox [1877, p. 30], after having described the "boomerangs" from Gujarat (see further down), states:

> An improved from of this weapon [..] is used by the Marawás of Madura, and some of these are much thinner than the boomerang of the Kolis [Gujarat], and in practice I have found them to fly with a return flight like the Australian boomerang.

However, Lane Fox gives no details at all concerning his throwing experiments, and he does not define what he means by the term "return flight", as was also remarked by Davidson [1935b, p. 175]. I might add that Lane Fox experimented with "fac-similes" rather than with the original implements themselves. (See also further down under Egypt.) The collection described by Lane Fox [1877] contains quite some fac-similes of boomerangs, e.g.:

> 179. Fac-simile of another Boomerang, thinner than the preceding and better adapted for flight. [Lane Fox, 1877,.p. 36]

Moreover, in 1883 Lane Fox (then named Pitt Rivers) says about the returning boomerang:

> This last kind of boomerang, I have contended, is merely a variety of the war boomerang, and is peculiar to the continent of Australia, and not found elsewhere, and that it is a development of the plain war boomerang, which latter is used by several of the black races bordering on the Indian Ocean as well as by the Australians. [Pitt Rivers, 1883, p. 457/8]

Boomerang-like objects have also been reported from another part of India, Gujarat. Walker [1924, p. 205] describes a wooden specimen:

> The kâtar is clumsily made and is only capable of travelling in an approximately straight line, it is about 26" [66 cm.] long, $1^3/_4$" [4.4 cm.] wide, $^5/_8$" [1.6 cm.] thick and weighs $9^1/_2$ oz. [270 g.].

Lane Fox [1868, p. 426, pl. XX] describes

> ... specimens of the "katureea" or boomerang from Goojerat, from the India Museum; they are used by the Koolees, according to the ticket in the Museum, "for whirling at hares, boars, and other wild animals, and disabling them." It is of raen wood, thicker and heavier than the Australian specimens, and therefore not adapted to rise in the air and return. The section is equal on both sides, but in other respects it is precisely identical with the Australian weapon, and appears to be roughly chipped into form.

Sinclair [1897, p. 79], referring to non-returning boomerangs, remarks:

> ... in British India at least one race, the Kolis of Northern Gujarat, have the like. These are invariably of "fish" section, varying in weight, curve, and material; but the commonest and most efficient sort is of "Babul" wood (*Acacia arabica*), with the natural curve of the heart of the wood, something like that of an old-fashioned Turkish sabre, rather a "knee" than any regular geometric curve.
> They are used with great effect on ground-game; much less of course, on birds.

These implements would seem to be throwing sticks rather than non-returning boomerangs.

Lommel & Lommel [1959, p. 160, fig. 70] reproduce a picture (copied from [Mitra, 1923, pl. X]) from a prehistoric cave painting at Singanpur (Raigarh, east central India), said by them to represent a human figure holding a boomerang. But it might just as well represent a man having both arms bent at the elbows and holding a shield and a stick.

Pictures of Indian "boomerangs" are published by Lane Fox [1868, pl. XX], [1877, pl. II], Egerton [1896, p. 73], Thurston [1907, pl. XXXVII], Walker [1924, p. 205], Hornell [1924, p. 337] and others.

7. Ancient Near East.

According to Bonnet [1926] (see also [Ebert, 1929, p. 450/1]) throw-sticks were widely used in the ancient Orient. There seems to be no evidence that these were anything more than curved sticks, although Bonnet [1926, p. 109] (on basis of doubtful data) distinguishes two types: throw-sticks with respectively round and flattened cross sections. See also [Petrie, 1917, pl. XLIII]. Bonomi [1852, p. 134/5] describes an Assyrian statue in Khorsabad as representing Nimrod, and the snake-shaped object in its right hand is said by him to be the analogue of the Australian "Bommereng". But it would be just as easy to take the object around Nimrod's left arm for a wrist watch. Nies [1914] provides evidence for the boomerang's being used in ancient Babylonia, in the form of a table showing the evolution of a cuneiform sign from a prehistoric pictograph resembling a boomerang:

> ... the sign we are considering, whose name is *gešpu*, whose values are *ru* and *šub*, and whose meanings point to the boomerang. [Nies, 1914, p. 31]

This evidence does not seem very convincing.

8. Africa.

The objects found in Africa south of Egypt which have been called boomerangs by many authors can hardly be more than throw-sticks, clubs or throwing irons. Most of these authors appear to be adherents of Kulturkreise theories, e.g. Ankermann [1905, p. 60], Schmidt [1910, p. 277/8], [1924, p. 82/3], Foy [1913, p. 249/50], Laviosa-Zambotti [1947, p. 151/4]. They apply the term "boomerang" or "Bumerang", rather easily it seems, to many objects of diverse kinds. Yet it is not impossible that some of these objects might in fact be simple forms of non-returning boomerangs.

An early description of the "trombash" of Sudan or Abyssinia is given by Baker [1867, p. 511]:

> There is a curious weapon, the trombāsh, that is used by these people [The Tokrooris] somewhat resembling the Australian boomerang; it is a piece of flat, hard wood, about two feet [60 cm.] in length, the end of which turns sharply at an angle of about 30°. They throw this with great dexterity, and inflict severe wounds with the hard and sharp edge; but, unlike the boomerang, the weapon does not return to the thrower.

Pitt Rivers [1883, p. 458, pl. XIV] gives

... illustrations of two of these wooden boomerangs, called trombush on the upper Nile [...]. It will be seen that they resemble some of the Australian boomerangs in form and section.

See also [Rütimeyer, 1911, p. 245] and [Bonomi, 1852, p. 135/6].

The Central African throwing irons, called trombash, kulpeda or pinga can have the most fantastic shapes, see for instance [Schurtz, 1889, Taf. V] and [Thomas, 1925, p. 136/7]. The blades of these weapons are in one plane. According to Schweinfurth [1875a, text to pl. XII]:

The "Pingah" is thrown in such a way as to turn in the horizontal plane round its axis, and by means of its three shanks, no matter in what position it reaches its aim, in every instance strikes with a sharp edge.

Lane Fox [1868, p. 429] compares these throwing irons to boomerangs:

In all, the principle of construction is the same, the divergent lateral blades serving the purpose of wings, like the arms of the Australian boomerang, to sustain the weapon in the air when spun horizontally.

But to classify such African weapons as boomerangs would be far-fetched indeed.

9. Ancient Egypt.

Of the regions outside Australia where boomerangs may have been in use the most interesting case perhaps is ancient Egypt. Many boomerang-like objects have been found, from the 6th, 11th and 18th Dynasties (respectively about 2300, 2000, 1400 B.C.), for instance. Egyptian wall paintings showing throw-sticks being used by fowlers are published by Wilkinson [1878, Vol. 2 p. 104, 107, 108], Erman [1885, p. 322], Trust. Brit. Mus. [1972, 48] and others. The depicted throw-sticks all have a slight S curve, whereas the specimens actually found have planforms resembling those of Australian boomerangs.

Fowling in the swamps of the Nile was a sport practised by the Egyptian nobles from early times. As a desirable occupation in the Next World it was represented on the walls of their tombs, the deceased owner being shown in the act of throwing his boomerang at birds rising from the swamp. [Trust. Brit. Mus., 1972, 48].

Wilkinson [1878, Vol. 2 p. 104/5] states:

The use of the throw-stick was very general, every amateur chasseur priding himself on the dexterity he displayed with this missile: and being made of heavy wood, flat, and offering little surface to the air in the direction of flight, the distance to which an expert arm could throw it was considerable; though they always endeavoured to approach the birds as near as possible, under the cover of the bushes or reeds.

And he remarks [Wilkinson, 1878, Vol. 3 p. 325 footnote]:

This calls to mind the boomerang of New Holland [= Australia]; but the peculiarity of this last, of coming back to the thrower, did not belong to the Egyptian throw-stick, which was also more straight.

However, there are at least three cases of authors claiming Egyptian boomerangs to be of the returning type. Erman [1885, p. 323] gives a lively description of an Egyptian fowler's boomerang hitting its target with much force and then returning in an elegant curve to the thrower. It is significant that in Erman's revised edition of 1923 this passage has been deleted [Davidson, 1935b, p. 177]. The second case is that of Lane Fox (again). In 1868 he describes "an ancient Egyptian boomerang of wood, in the British Museum" as follows [Lane Fox, 1868, p. 427]:

It is of hard but light wood, the section is symmetrical on both sides, and not flat on one side, like some of the Australian boomerangs; it is somewhat broader at the ends than in the middle of the blade.

In 1872 he writes [Lane Fox, 1872, p. 324]:

I have practised with the boomerangs of different nations. I made a *fac simile* of the Egyptian boomerang in the British Museum, and practised with it for some time upon Wormwood Scrubs, and I found that in time I could increase the range from fifty to one hundred paces, which is much farther than I could throw an ordinary stick of the same size with accuracy. I also succeeded in at last obtaining a slight return of flight; in fact it flies better than many Australian boomerangs, for they vary considerably in size, weight and form, and many will not return when thrown.

In 1877 the "slight return" has become "returning to within a few feet" [Lane Fox, 1877, p. 31]:

In order to ascertain by experiment whether this was really a boomerang, I had these fac-similes made with great care from the original of different kinds of wood, and they have been found by experiment to fly like a boomerang, ranging about 100 paces, and returning to within a few feet of the thrower. This experiment settles the question of the use of the boomerang by the Egyptians, which, owing to the ill-defined representations of them in Egyptian sculptures, was previously open to dispute.

But in 1883 the same author (as was mentioned earlier under India) states that returning boomerangs are "peculiar to the continent of Australia, and not found elsewhere" [Pitt Rivers, 1883, p. 457]. It seems not unlikely that the experiments reported by Lane Fox were biased.

The third case of Egyptian boomerangs reported to be of the returning kind is published by Hayes [1953, p. 284] of the Metropolitan Museum of Art in New York:

> Two examples from tombs of the Eleventh Dynasty at Thebes are of the so-called "return type" - wooden blades, flat on one side and slightly convex on the other, curved to an angle of about 140°, with a rounded handle end and a blunt hitting end slightly broader than the rest of the blade [...]. An exact reproduction of one of these weapons was found, when cast, to fly out a long distance, make a sharp turn, and come back to the thrower. The boomerang so tested was made to be thrown with the right hand, and this of course was usually the case, although similar weapons for left-handed hunters are also known.

Unfortunately no details are available as regards the way the copy was made and thrown. The records of the department of Egyptian art of the Metropolitan Museum show that

> ... one of our boomerangs was copied sometime in 1930 or 1931 (probably here in the Museum workshops), but the manner in which it was copied is not known. The description of the flight of the model is as follows: "long cast, sharp turn, and will return to thrower." There are no further notes regarding experiments with this or other boomerangs. [Miss V. VonderPorten, personal communication, 1973]

It would be worthwile to carry out new experiments with very carefully made replicas of ancient Egyptian boomerangs.

A very essential feature of any boomerang is its cross section. Pitt Rivers (= Lane Fox) [1883, pl. XIV] presents drawings of several Egyptian boomerangs with cross sections. However, these are too small to show much detail, although the author has

> ... shown clearly by the sections attached to each that they are true flat boomerangs, and not merely round curved sticks... [Pitt Rivers, 1883, p. 457]

Nies [1914, p. 28/9] describes a boomerang of the "XVIII Dynasty or earlier" as follows:

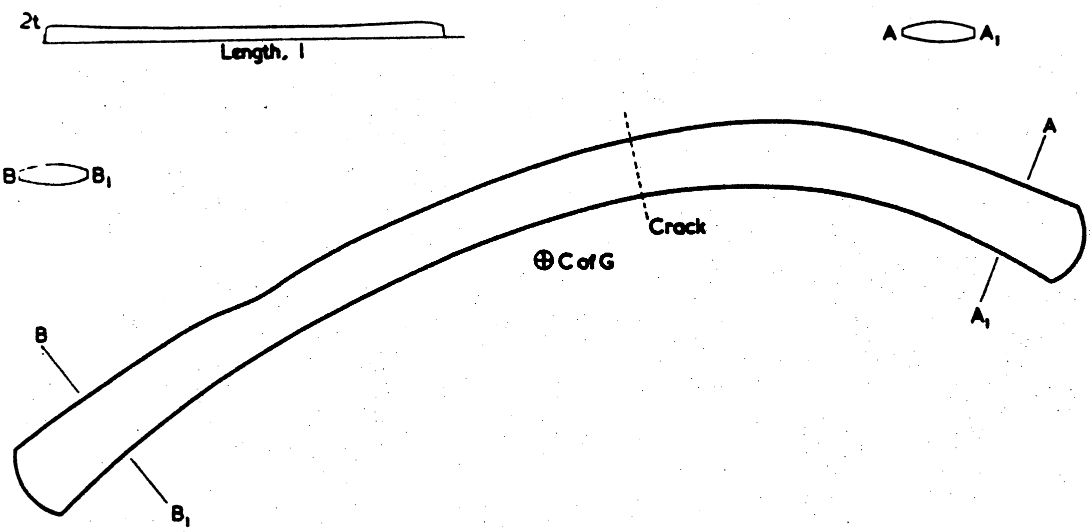
> It is slightly flat on one side, convex on the other, and has a rather wide angle [...]. It weighs 6 ounces [170 g.], is 4.3 cm. wide, 1.3 thick

at the middle, 54 cm. round, 46 cm. across, and the arch is 13.5 cm. high. The angle I have not taken. [...]. It does not seem to have the skew or elevation of 2° to 3° at the points, which Thomas [1910] states are necessary to give the weapon its peculiar flight.

Pictures of boomerangs found in the tomb of Tutankhamen (~ 1350 B.C.) are published by Carter [1933, pl. LXXVI, LXXVII] and Desroches-Noblecourt [1963, p. 271]. Carter [1933, p. 142] describes these objects as follows:

Boomerangs and throw-sticks were used in Egypt from the earliest to the last dynasties. The boomerang was certainly used for fowling; the throw-stick probably in warfare. Both kinds are represented in this collection. Of the first type among this lot - boomerangs proper - the return and the non-return kinds are recognizable, even though the general form of both weapons is much the same, i.e. curved in sickle-shape, or two straight arms at an angle, the main, or rather the essential, difference being the skew (twist) of the arms, which are exactly opposed in the two kinds. The non-return weapon was apparently thrown like the return type, its reverse twist or skew helping it to travel a greater distance than the ordinary throw-sticks.
Our specimens of boomerangs are made of a hard wood which I am unable to recognize; they are either painted with a polychrome pattern, or bound in part with a bark resembling that of the birch tree [...]. The ritualistic specimens are of carved ivory, mounted with gold caps at the ends.
The throw-sticks here are either of fantastic form [...] or of simple curved shape made of a hard wood. Those made of ebony with ends of gilt are probably ritualistic, like the example made of gilt-wood capped with faience, or those solely made of faience.

It is remarkable that the boomerangs generally are wider at the ends than in the middle. Many of them are partly covered with bark. Howard Carter's original (unpublished) descriptions are accompanied by sketches of planforms as well as cross sections. Some of Tutankhamen's boomerangs have symmetrical biconvex sections, but others have asymmetrical or even plano-convex sections. The measurements of 18 of the wooden boomerangs as described by Carter in these unpublished notes vary within the following limits: tip-to-tip length 26.5-64.0 cm., width at the centre 2.8-5.2 cm., thickness at the centre 0.55-1.5 cm., thickness over chordlength ratio at the centre: 0.20-0.42. (Median values resp. 50 cm., 3.8 cm., 1.1 cm., 0.29). Carter's very interesting data (as well as other material relating to Egyptian boomerangs, see fig. 10.2) are to be published in the near future by Mr. V. Davies, Griffith Institute, Ashmolean Museum, Oxford, and Dr. P.J. Musgrove, Department of Engineering and Cybernetics, University of Reading (England).



EGYPTIAN 6th DYNASTY c.2300 B.C. Brit. Mus. No. 45213

Length 715mm; Av. Width 39mm; Av. Thickness 13mm; Weight 200g; Planform Camber 15·7%;
Radius of Gyration 217mm; Incidence, Right Wing Tip + $4\frac{1}{2}^{\circ}$, Left Wing Tip − 1°.
Dihedral, Indeterminate due to crack.

fig. 10.2 Egyptian boomerang;
measurements and drawing by Dr. P.J. Musgrove.

From the available literature it appears that ancient Egyptian boomerangs can have various cross sections, often not unlike those of Australian boomerangs. Therefore it seems possible that some of them are of the returning kind, and purposely made to be such, whereas some others may be non-returning or even straight-flying boomerangs. Further research on the flight properties of ancient Egyptian boomerangs would be extremely interesting!

10. Ancient Europe.

Ferguson [1838a] was the first to study non-Australian boomerangs. His learned philological paper is mainly concerned with examining classical writings in order to find evidence for the use of the boomerang in ancient Europe. It is apparently the only study of its kind. The strongest case of a boomerang-like implement certainly is the cateia, mentioned by Virgil, Silius Italicus and Valerius Flaccus [Ferguson, 1838a, p. 22/3]. From these passages the cateia appears to be some sort of throwing stick or club. The opinion that the cateia could be a returning boomerang rests on a description by Isidore, Bishop of Sevilla (Spain) in his encyclopaedia (~ 624 A.D.):

> He describes the Cateia as a species of bat, of half a cubit in length, which, on being thrown, flies not far, on account of its weight, but where it strikes, it breaks through with excessive impetus. *And if it be thrown by one skilful in its use, it returns back again to him who dismissed it.* The passage occurs in the "Origines," under the head CLAVA, viz.:
> "CLAVA est qualis fuit Herculis, dicta quod sit clavis ferreis invicem religata, et est cubito semis facta in longitudine. Hæc et Cateia, quam Horatius *Caiam* dicit. Est genus Gallici teli ex materia quam maxime lenta; quæ, jacta quidem, non longe, propter gravitatem, evolat, sed ubi pervenit vi nimia perfringit. *Quod si ab artifice mittatur, rursum redit ad eum qui misit.* Hujus meminit Virgilius dicens
>
> 'Teutonico ritu soliti torquere Cateias.'
>
> Unde et eas Hispani *Teutones* vocant." - *Isidor. Origin.* l. xviii. c. vii. Thus, all the characteristics of the Boomerang, its use, its shape, its mode of projection, its extraordinary impetus, and its peculiar reciprocating flight, belong to the Cateia, from which it cannot but be concluded that these were the same weapon. [Ferguson, 1838a, p. 23/4]

This passage by Isidore has been quoted by many others, e.g. Lane Fox [1868, p. 430], Burton [1884, p. 35], Franz [1928, p. 805/6], Feldhaus [1931, p. 230], Lenoch [1949, p. 51]. But is this clava, cateia or caia really a returning boomerang? A different interpretation is possible. The weapon could very well be only a throwing club made "of extremely tough wood" ("ex materia quam maxime lenta"). It "flies not far because of its weight", which indicates a weapon for fighting at rather close quarters. That it resembles the club of Hercules makes its boomerang behaviour even less likely, instead of making Hercules' clava a boomerang as well [Ferguson, 1838a, p. 34/7]. It seems utterly improbable that such a heavy club-like weapon would be able to return as a boomerang. If it would return at all this could be due to its bounding back, from the target, after being "launched by a skilful man". There seems to be nothing in Isidore's description against this interpretation. Ferguson's identification of various other weapons (e.g. aclys, ancyle) with returning boomerangs is even more improbable, and to a great extent based on ethymological speculations. Thor's mythological hammer is supposed by Ferguson [1838a, p. 37/8] to be a T-shaped returning boomerang, and many more curious inferences are drawn in Ferguson's paper.

Lehmann-Nitsche [1936] mentions the use of throw-sticks by the Greeks, Etruscans and Romans, in (ritual) rabbit and hare hunting. But these rabbit-sticks probably are not boomerangs. Thus it seems that the evidence for (returning or non-returning) boomerangs in ancient Europe is extremely weak.

11. Prehistoric Europe.

Although there is only scarce evidence for the presence of boomerangs in prehistoric Europe, it seems to be sufficient for several authors to draw far-reaching conclusions from it. Mostly these authors are adherents of Kulturkreise theories, e.g. Franz [1928], Laviosa-Zambotti [1947], Narr [1952]. As an example consider the following passage from a paper called "Alteuropäische Wurfhölzer", which is fairly typical:

> Von den Kulturkreisen der Ethnologie führt den Bumerang als typisches Gut die exogam-gleichrechtliche (= Bumerangkultur), die sich am verhältnismässig stärksten in der Südsee (Australien) und in bestimmten Teilen Afrikas (Nilquellgebiet) erhalten hat; doch ist der Gedanke des Wurfholzes von seinem alten Zentrum aus weit gewandert, wie die heutige Verbreitung beweist. In vorgeschichtlicher Zeit hat er sich besonders in Vorderasien breit gemacht, ferner in Afrika [...]. Während sich in letzterem Gebiete aus dem Wurfholz das Wurfmesser entwickelt hat, ist in Vorderasien aus ihm ein metallener Krummsäbel entstanden [...]. Wenn mann unsere Fig. 10, eine Kupferwaffe aus Babylonien ansieht, so fühlt man sich sofort an das Holzstück Fig. 9 aus dem Brabandsee erinnert; nur ist der kürzere Teil bei der fortgeschrittenen Metallwaffe an der Spitze nach oben gebogen, ein Zug, der jedoch die enge typologische Verwandtschaft nicht zu verschleiern vermag. [Franz, 1928, p. 802]

The terms "Bumerang" and "Wurfholz" are interchangeably used, and a distinction between returning and non-returning boomerangs, throw-sticks and even curved swords seems to be unimportant to some Kulturkreiseminded authors:

Ob es sich dabei tatsächlich um Bumerangs sensu stricto, also um Kehrwiederkeulen handelt, ist natürlich nicht mehr fest zu stellen, aber auch relativ unwichtig, da die Wurfkeule für Jagd und Krieg auch in Australien nicht dieses Prinzip verkörpert. [Narr, 1952, p. 1020 footnote]

Indeed, the extant evidence often is so meagre that it does not allow to asses the function and use of a prehistoric boomerang-like object.

The evidence is of two kinds: cavepaintings and finds of actual objects. A cavepainting in Niaux is said by some to contain representations of boomerangs [Sollas, 1911, p. 234/5], but there is hardly even a superficial resemblance. The same applies to a picture from Pindal, reproduced by Schmidt [1934, p. 114]. The cavepaintings in Minateda, as published by Breuil [1920], contain several little curved figures, but, in contrast to the depicted bows-and-arrows, their use is not at all clearly indicated. The only rather convincing European "boomerang" cavepainting is the one reproduced in [Kühn, 1952, Taf. 46] (copied in [Lommel & Lommel, 1959, p. 160]); it comes from Albarracín (Spain). But of course the "boomerangs" represented in this rockpainting could just as well be mere curved throwing sticks.

A most remarkable find of wooden objects resembling boomerangs is that of Braband Sø (Denmark, ~ 4000 B.C.) [Thomsen & Jessen, 1906a,b]. These objects are remains from the Ertebølle culture. Of 16 peculiar wooden objects one in particular might be a sort of boomerang. See fig. 11.1.

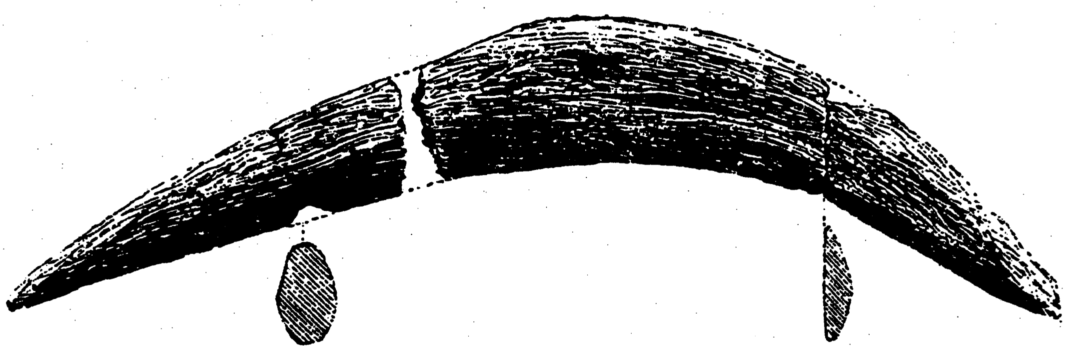


fig. 11.1 "Boomerang" from Braband Sø, ~ 4000 B.C.
(copied from [Thomsen & Jessen, 1906a, p. 42])

It is described in detail by Thomsen & Jessen [1906a, p. 41/2] = [1906b, p. 199/200]. The implement is made of maple wood, its tip-to-tip length is 41.5 cm., its greatest width 5.5 cm. It tapers toward both ends, which are pointed. One arm has an oval cross section (thickness/chord $\approx$ 0.58), the section of the other arm is flattened and plano-convex (thickness/chord $\approx$ 0.26). When thrown as a boomerang (in its original condition), it would undoubtedly have flown better than a mere throw-stick, but it probably would not have returned. This implement could be a non-returning boomerang, although it is difficult to understand why only *one* arm has a flattened cross section. Several of the other wooden objects found at Braband Sø might be throw-sticks, but none have cross sections suitable for boomerangs. See [Thomsen & Jessen, 1906a, p. 43/7] = [1906b, p. 201/5]. (Franz [1928, p. 800/1] is eager to take all these wooden objects, including the straight ones, for throw-sticks).

Schoetensack [1901, p. 138/40] describes two curved, 8-11 cm. long, palaeolithic objects cut from reindeer antler, found in Dordogne (France). These are considered by some to be toy boomerangs. Rütimeyer [1911, p. 245] mentions a Copper Age "Wurfholz" found near the Bieler See (Switzerland). His accompanying figure shows an axe-shaped wooden implement. Schwabedissen [1951, p. 309] represents several wooden objects from Schleswig (Germany), these do not at all resemble boomerangs. Narr [1952, p. 1020] sees boomerangs and/or throw-sticks spread all over Europe, he supports his opinion by referring to [Breuil, 1920], [Franz, 1928], [Schmidt, 1934] and [Schwabedissen, 1951].

Müller-Beck [1965, p. 55/6] describes some wooden objects from Seeberg and Egolzwil (Cortaillod culture) in Switzerland. One such object consists of a straight flat part with rectangular cross section joined at a right angle to a straight handle of circular cross section, the overall shape resembling that of an axe. Müller-Beck [1965, p. 56] remarks that it is suited to be used as a striking implement, and he tentatively suggests the possibility of the object's having been used as a throw-stick. But its peculiar form impresses one as being carefully designed for some specific purpose (unknown to me), whereas it is not a form one would expect for a carefully designed throw-stick or boomerang.

Finally, an oak boomerang from the Iron Age (~ 300 B.C.) has been unearthed near Velsen (Netherlands). Pictures of it are published by Calkoen [1963a, p. 74] and Hess [1973b, pl. XLIV].

The distance from tip to tip is nearly 39 cm., the arms are 3.2-3.7 cm. wide and 0.7-0.9 cm. thick. In the centre part of the bend the thickness at some points is 1.0 cm. The angle between the arms is slightly greater than 110°. The grain of the wood follows the general shape of the object. [...].
The boomerang appears to be carefully worked. One side, which I call the upper side [...] is convex, the other side [...] is less convex or flat, and even slightly concave at one end. Probably all of the notches to be seen on this side were made during the excavation. The boomerang's surface may have been smooth originally. The arms of the boomerang do not exactly lie in one plane, and they are twisted near the ends. Probably the boomerang was designed flat, with perhaps a slight twist at the ends. [Hess, 1973b, p. 304]

Soon after the boomerang was found in 1962, Mr. M. Ingen Housz made a plywood copy with which he was able to obtain return flights [Calkoen, 1963b, p. 37], but no details were published. This copy was made after Calkoen's drawing rather than based on detailed measurements of the original object itself [M. Ingen Housz, personal communication, 1972].

The original boomerang unfortunately was badly deformed during preservation. However, excellent moulds of each of its three pieces are kept at the Rijksmuseum voor Oudheden at Leiden, Netherlands. From these negatives a positive cast (epoxy resin) was made. [Hess, 1973b, p. 303/4]

In order to investigate the flight properties of the Velsen boomerang, I made a plywood copy, based on the epoxy cast, which could be used in field experiments:

The contour of the copy was sawn out of 0.9 cm. eight-layered birch plywood. With a file the arms were carefully shaped so that the thickness of the copy matched that of the cast [fig. 11.2a] at some 60 measured points. The upper and underside were carefully made to resemble those of the cast, but the surfaces were sanded smooth and the arms were kept in one plane, so that the probable warp of the object was corrected. However, some twist at the ends was provided [i.e. retained]. During the throwing experiments the mass of the plywood model was approximately 72 g, its density 0.73g/cm^3. [Hess, 1973b, p. 304]

The mass of the original oak boomerang may have been somewhat greater or smaller.

The arms of the copy (like those of the original) have cross sections

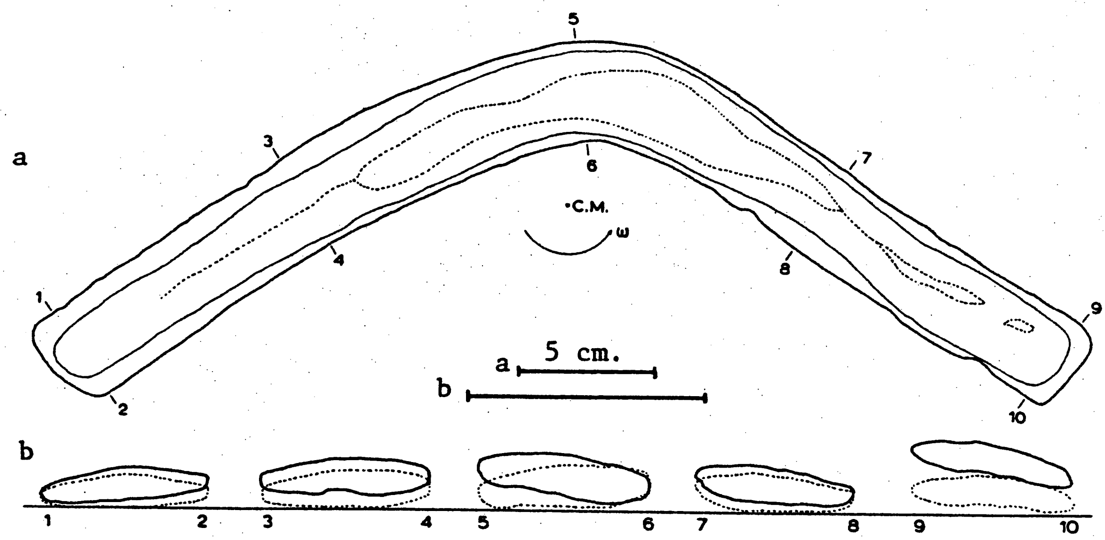


fig. 11.2 Boomerang from Velsen (~ 300 BC).
a) Thickness map of boomerang shown from the upper side.
Thin drawn line: thickness = 0.6 cm., dotted line: thickness = 0.8 cm.
CM: centre of mass. ω: direction of boomerang's spin.
b) Cross sections cut at places marked by nos. 1-10. Horizontal line: plane of support. Solid lines: present state of sections, dotted lines: sections obtained after correction for probable warp of original.

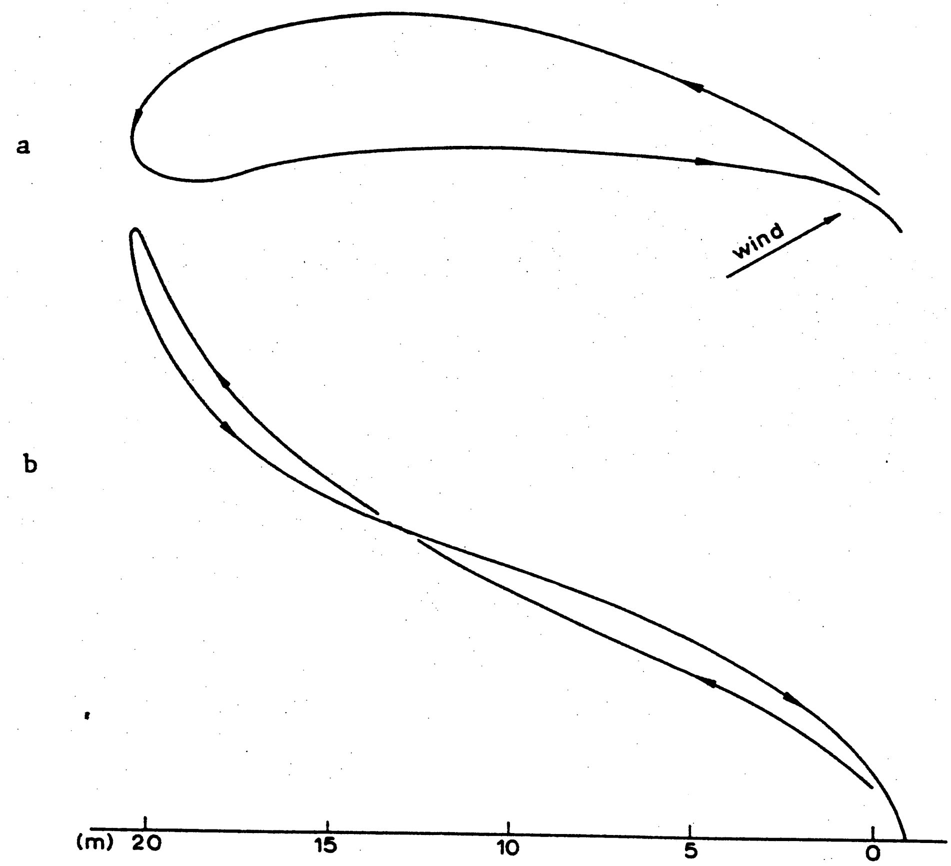


fig. 11.3 Typical return trajectory traversed by the plywood copy.
a) Bird's-eye view, b) side view.

[fig. 11.2b] which would not seem very favourable for airfoils: a thickness of more than 20 per cent of the chord length, no smooth leading edge, low Reynolds numbers (Re < 10^5). Nevertheless the copy was capable of completing return trajectories. Its spin, however, visibly decreased during the flights because of the airdrag [...]. A typical return trajectory completed by the copy has been sketched [in fig. 11.3].
The throwing experiments were done in the open air on grassland. The boomerang was thrown righthandedly in a direction of 20°-30° upward. The angle between the boomerang's plane of rotation and the horizon was 70°-80°. The greatest distance, measured horizontally, between boomerang and starting point was 20-21 m. The highest point of the trajectory was 15-18 m. above ground level, as determined with tape measure and protractor. (Lower flightpaths were possible, but only as open loops. Perhaps with strong wind low return trajectories could be realized.)
The duration of the flight was 6-7 seconds. The initial forward and rotational velocities were not measured; my rough estimates are 25 m/s and 13 rev/s respectively.
The wind speed, measured at 2 m. above ground level, was very low during the experiments; it varied between ½ and 1m/s (probably the wind was somewhat stronger at greater heights). The boomerang was thrown some 60° to the right of windward. [Hess, 1973b, p. 304]

It should be remarked that it was not very easy for the thrower (H. Rollema) to make the boomerang return completely. When not thrown carefully and at high speed, it would describe only something like a half circle. The following conclusion may be drawn:

The plywood copy thus behaves like a right-handed returning boomerang of moderate quality. Naturally the original boomerang may have differed somewhat in shape and mass from my own copy and hence may have had flight-paths of different shape. But it is very probable that it was capable of returning to its thrower if properly thrown. Possibly it had first to be adjusted over a fire when it was warped. [Hess, 1973, p. 305]

(As to the methods of adjusting boomerangs see §5). That the Velsen implement was *actually* used as a returning boomerang is not proven however, only that it *could* have been used as such. It is remarkable that

The boomerang could also be made to fly a distance of some 50 m. along a nearly straight horizontal line, by throwing it left-handedly (opposite spin), with its plane of rotation approximately horizontal. [Hess, 1973b, p. 304]

Unfortunately the Velsen boomerang is an isolated case, it would stand up much stronger if it would be corroborated by similar prehistoric finds. However, prehistoric wooden objects are rarely preserved. Even if boomerangs would have been widely used in Europe, only a tiny fraction of them would eventually be found by archaeologists.

Conclusion.

Most of the available relevant data on possible non-Australian boomerangs have been mentioned in §§9, 10, 11. A more comprehensive survey of the literature is given by Lenoch [1949] in his dissertation "Wurfholz und Bumerang". He attempts to discuss *all* publications concerning throw-sticks and boomerangs, from whatever age or place on earth. (His bibliography contains some 550 titles, but part of these are not relevant to the subject). But although Lenoch's study constitutes an admirable collection of otherwise scattered data published by others, it hardly offers new insights. Also Lenoch does not try consistently to distinguish between throw-sticks and boomerangs, which admittedly would not always be a simple matter. A shorter, but critical survey is given by Davidson [1935b, p. 169-180]. Lommel & Lommel [1959, p. 161] draw a world map showing "die Verbreitung des Bumerangs". But the indicated regions are those in which throwing sticks have been found, rather than true boomerangs.

It seems likely that sticks have been thrown almost all over the world, at any time. Implements specially adapted for being thrown (throw-sticks, throwing clubs etc.) also are, or have been, extensively used in various regions. But as regards boomerangs the evidence is scanty, except, of course, for Australia. Special types of throwing implements were in use on the New Hebrides, among the Pueblo Indians, in India, north Africa and prehistoric Europe. But most of these would be border cases rather than real (non-returning) boomerangs. In ancient Egypt possibly non-returning as well as returning boomerang were used, but more research is necessary to settle the question. One of the Braband Sø implements might be a kind of non-returning boomerang. The Velsen boomerang is one, hitherto isolated, case of a returning boomerang in prehistoric Europe. The tela and cross boomerang of Celebes are returning implements, but only used as children's toys. Although they are a class apart, technically they should be termed boomerangs.

A final remark might be made about the "European" boomerangs produced - in Australia, Europe, America, Japan, etc. - ever since the returning boomerang became popular in Dublin around 1837 (see [Dubl. Univ., 1838]).

Boomerangs are being increasingly made and sold overseas. London toyshops feature "safe" boomerangs for children. These are small and made of balsa. Various wooden and plastic versions are on sale in America, including a model for children, a sports version for women, and a heavy one for men. Another model is patriotically fashioned in the style of an American eagle with outstretched wings and is called the "Boomer Bird." Japan and France are making and selling boomerangs on the home market. The Japanese versions, complete with instructions for throwing, are sleek, a foot long, and of plastic in gay colours. The French luxury model is of aluminium and has special finger grips and padding on the points. [Stivens, 1963, p. 10]

Although these boomerangs are ultimately derived from Australian Aboriginal boomerangs, they may be considered as typical products of the "Western Civilisation". Sometimes modern aerodynamic knowledge is incorporated in their design, new materials are used (plywood, metal, plastics, etc.) and often they are mass-produced. Also new forms may be tried (see for instance [Mason, 1937]). All of them are meant to be truly returning, but many of the commercially sold boomerangs are failures in this respect. One example:

The Brist Boomerang, with which Brist is played, will go down in history as the most wonderful novel invention of the age. If the following instructions are carefully observed, anyone can, with but little practice, do with it such incredulous things, that were you to have appeared among your friends for half an hour's entertainment, a century ago, you would duly have been tried and convicted of witchcraft. [Bristow, 1912]

A special class is formed by the boomerangs designed and thrown by individuals either for fun or sport or to satisfy their scientific curiosity.

§12 *The origin of boomerangs.*

"How were boomerangs invented?" or "How could the improbable idea of making such an extraordinary implement have occurred to the Aborigines?" This question inevitably turns up when somebody for the first time watches a returning boomerang fly through the air. A somewhat different question is whether Australian boomerangs originated in Australia or whether they were imported there from other parts of the world. To both questions the answers are unknown.

When the first Europeans arrived in Australia, boomerangs were already there. Dampier, who visited the coast of Western Australia in 1688, may have been the first to mention boomerangs:

> Some of them had wooden Swords, others had a sort of Lances. The Sword is a piece of Wood shaped somewhat like a Cutlass. [Dampier, 1729, p. 314]

Cook, who landed in Botany Bay in 1770, writes about the Aborigines he encountered there:

> ... all of them were armed with long pikes, and a wooden weapon shaped somewhat like a cimeter [= scimitar].
> ... each of these men held in his hand the weapon that had been described to us as like a cimeter, which appeared to be about two feet and a half long ... [Cook, 1773, p. 491]

The only Australian boomerang radiocarbon dated so far appears to be the one found in the gravel beds of the Clarence River near Grafton. The radiocarbon date is 140 ± 60 BP (Gak 1299), but since there are possibilities of contamination, it is best to regard the age of this specimen as "No earlier than 1550 A.D.". [Dr. Isabel McBryde, personal communication, 1973].

Very recently (January 1974) some exceedingly ancient boomerangs have been unearthed by Mr. R. Luebbers from a peat bog in south east South-Australia [Luebbers, 1975]. No radio carbon dating has been done as yet, but probably these implements stem from 9000 B.C. or earlier.

> Asked how the conclusion that the boomerangs were at least 11,000 years old was determined, Mr. Luebbers explained that a peat layer some 50 centimetres (cm) above the artifact-bearing level had been radiocarbon dated at 9000 years.

"We expect the dates of the boomerangs to be at least 2000 years older," Mr. Luebbers said. "Our knowledge of the prehistory of the area indicates the implements would not be any older than 15,000 years, because at that time it was too dry and peat could not have formed." [Bransdon, 1974, p. 2]

Mr. R. Luebbers (personal communication, 1974) adds:

... at least three boomerangs were recovered complete although only one was intact. It measures 39 cm across and is 2.75 cm wide - these measurements taken while the wood is still wet. An additional four boomerangs seem to be represented by wing tips but all of these are in pieces at the moment. So I can provide mere impression and not facts. In general each specimen displays aerofoil design features of two broad types; 1) thickly ovoid with thickness three quarters of width and 2) less thick, broader wings with flat undersides. Spans probably do not exceed 45 cm. Quite clearly the upper surfaces are more convex. It would be pointless however for me to describe wing characteristics further until the specimens are dried and reconstructed. We expect this to occur in six months time, at which time proper photos can be taken and experimental flight models made.

This extremely interesting find at once shifts the known age of Australian boomerangs far back in time, even beyond the Danish and Dutch prehistoric boomerangs.

Representations of boomerangs in Aboriginal rock art (see [McCarthy, 1958b, 1960a]) also indicate their old age:

Both returning and non-returning types are shown in rock engravings in the outline, Linear design, and Fully Pecked phases [...] which indicate that the boomerang is quite ancient in Australia, and in rock paintings in various phases of local sequences. It is interesting to note that returning boomerangs are painted in rock shelters on Groote and Chasm islands, in the Gulf of Carpentaria, where they are no longer in use. None of these paintings or engravings has been radiocarbon dated. [Mr. F.D. McCarthy, personal communication, 1973]

The oldest reliably dated case of Australian rock art, without representations of boomerangs however, is about 20,000 years old [Edwards & Ucko, 1973, p.275]

Did the first Australians take the boomerang (or a prototype) with them when they came from South East Asia, or did they import the boomerang at a later time? Or was the boomerang developed in Australia (and perhaps in some other parts of the world as well)? Lommel & Lommel [1959, p. 158/9], for instance, take a diffusionist standpoint:

Australien hat offensichtlich sehr frühe und altertümliche Kulturformen und Geräte bis in verhältnismässig junge Zeit bewahrt. Zu den

bemerkenswertesten Geräten früher Epochen, die sich in Australien erhalten haben, gehört der Bumerang und die Speerschleuder. Beide Geräte finden sich schon in paläolithischen Höhlen Südfrankreichs.
Lenoch [1949] hat das ausseraustralische Vorkommen des Bumerangs im vorgeschichtlichen Spanien, in Nordafrika, in Indonesien und Melanesien und verschiedenen Gebieten Amerikas nachgewiesen.
Die Speerschleuder findet sich wieder im östlichen Asien und Nord-, Mittel-, und Südamerika. Es scheint, als ob sie einen nördlichen, der Bumerang einen südlichen Weg bis zu den Küsten des Pazifischen Ozeans genommen habe. Beide Strömungen könnten sich in Australien vereint haben.

This argument is poorly supported by data, see §11. Davidson [1935b, p. 167] is of a quite different opinion:

On the basis of Australian evidence, there seems to be no good reason for believing that boomerangs are not indigenous to Australia. In this instance the geographical distribution of boomerangs as a class appears to be illuminating. As we have already seen, both ordinary and returning boomerangs are lacking in the extensive area comprising the three northern peninsulas. It is presumable that they have never been used in the northern Kimberley district and northern North Australia, for we find the ordinary varieties diffusing into these areas at the present time. The crucial place where boomerangs seem to be unknown, but where we would expect to find some traces of them if they had been brought into Australia from a foreign source, is the Cape York Peninsula. For this region, Roth informs us that they are lacking north of the Palmer and Mitchell rivers.

I am inclined to agree with Davidson that boomerangs probably are indigenous to Australia. Why would the "invention" of the boomerang be easier, or more probable, outside Australia than inside? This brings us to the question posed right at the beginning of this section: "How were boomerangs invented?". A reasonable guess is made by Davidson [1935b, p. 168]:

In a culture where throwing-sticks undoubtedly have been in use for a great period of time, we do not have to look far for a possible and most reasonable ultimate basis from which boomerangs could have been derived. It should not be implied that there was necessarily a direct change from a throwing-stick to a boomerang by the reduction in height of the cross-section and the giving of a greater curvature, although such could have been and may have been the case. It seems much more reasonable to suppose that such a change, if it actually happened, was gradual and that considerable time may have elapsed before what we recognize as a boomerang was produced.

Essentially the same idea was put forward by Lane Fox [1868, p. 425]:

Instinct prompts him to eat, little better than instinct would enable him to select from amongst his weapons such as are found most suitable for obtaining food, and we have already seen how he may have been led to the adoption of such an instrument as the boomerang, purely through the laws of accidental variation, guided by the natural grain of the material in which he worked.

In some more detail [Lane Fox, 1877, p. 29]:

A curved stick, when thrown from the hand rotates of its own accord, and it would soon be discovered that a flat curved stick formed by splitting a branch in half down the centre would fly further than a round one. The savage would be entirely ignorant of the reason for this [...]; but he would find in practice that the thinner he made it the further it would fly, and this really consitutes the generic characteristic of the boomerang ...

And as regards returning boomerangs:

Finally, it would be discovered that when the boomerang was slightly twisted in a particular direction, like the two arms of a windmill set in oblique planes, it would screw itself up in the air. But this he would arrive at more probably by imperfect workmanship, owing to the difficulty of constructing the weapon on a true plane than from any knowledge of its principle of action. [Lane Fox, 1877, p. 30]

Apart from the supposed necessity of the twist in returning boomerangs (see §4), this seems to be no unreasonable picture. Sutton [1912, p. 218] puts it slightly differently:

Maybe one of the remote ancestors of the tribe, leaving his "war boomerang" out in the dew one night, found a day or two later that the flight had changed from a straight path to a more or less circular one; and probably his sporting instincts and curiosity developed the idea, chiefly for amusement, perhaps also for use in hunting and sport.

And Davidson [1935b, p. 168] says:

In respect to returning boomerangs, there seems to be no reason for doubting, on the basis of Australian evidence, that they have been derived from ordinary boomerangs somewhere in Australia. They, too, are not only lacking in the same northern peninsulas, toward which boomerangs are now diffusing, but their origin can be explained most logically in the similar non-returning boomerangs with which they are always associated.

Extensive experiments relevant to the evolution of straight-flying boomerangs are reported by Callahan[1975]. Hardly more can be said at present on the origin of boomerangs, but the curious, quite different, eucalyptus leaf hypothesis should be mentioned:

Mr. Hubert de Castella has suggested that the Aborigines derived the invention of the *Wonguim* from observation of the shape and the peculiar turn of the leaf of the white gum-tree. As the leaves of this tree fall to the ground, they gyrate very much in the same manner as the *Wonguim* does; and if one of the leaves is thrown straight forwards, it makes a curve and comes back. Such an origin for a weapon so remarkable is not to be put aside as unreasonable. It is very probable that if children played with such leaves, some old man would make of wood, to please them, a large model of the leaf, and its peculiar motions would soon give rise to curiosity and lead to fresh experiments. [Smyth, 1878, p. 316]

See also [Campbell, 1882, p. 460], [Lumholtz, 1889, p. 52], [Lenk-Chevitch, 1948, 1949], [Williams, 1952]. Campbell Ford [1913, p. 117] describes an Aboriginal game, watched in 1883:

Later, piling a lot of bushes on the fire to make a big flare, they started playing "Bindjhera," a game in which the dead leaves of the brigalow (Acacia harpophylla) were made into miniature boomerangs, and flipped with a rotary motion into the current of hot air, where, spinning with increased velocity, they climbed up and up in a beautiful spiral until they lost the influence of the draught and fluttered dejectedly down to mother earth again.

CHAPTER II

BOOMERANGS FROM A PHYSICAL VIEWPOINT

§13 *The behaviour of returning boomerangs.*

Boomerangs are objects which, after being properly launched, fly rapidly spinning through the air and return to the vicinity of the launching point. (Boomerangs of the non-returning and straight-flying kinds are not considered in this section.) This curious behaviour, so unexpected for anyone watching it for the first time, should be completely explainable in terms of the laws of physics. An elementary explanation of the returning boomerang is given in §16. The present section is descriptive.

A typical returning boomerang is thrown with its plane vertical (or slightly inclined with its upper part away from the thrower), in a horizontal (or slightly upward) direction, and with a considerable spin. At first the boomerang just seems to fly away, but it soon swerves to the left and also upwards, traverses a wide loop, approaches the thrower, and may descend somewhere near the thrower's feet, or describe a second, smaller, loop before reaching the ground. Generally, the boomerang's plane of rotation gradually "lies down", so that it may be nearly horizontal at the end of the flight. It is a splendid sight if a boomerang, quite near again after describing a loop, loses its forward speed, hovers some 5 metres above your head, and slowly descends like a helicopter or a maple seed.

Descriptions, however, can give only a rather poor idea of what a real boomerang flight is like. One should stand in the open air to see how very three-dimensional this phenomenon is and hear the soft, pulsating, swishing sound of the boomerang arms moving rapidly through the air. [Hess, 1968a, p. 126]

A naive observer might gain the impression that the boomerang is in the air for half a minute or so, whereas the real duration of the flight is typically about 8 seconds. Longer times are possible: the record duration of a boomerang flight witnessed by me was 22.0 seconds. In this case the boomerang was made and thrown by Mr. Allan Grantham in Reading

(U.K.), 29 June 1973. The timing was done independently by two observers using stopwatches (without the thrower's being aware of it). There was a light breeze and the boomerang returned fairly well.

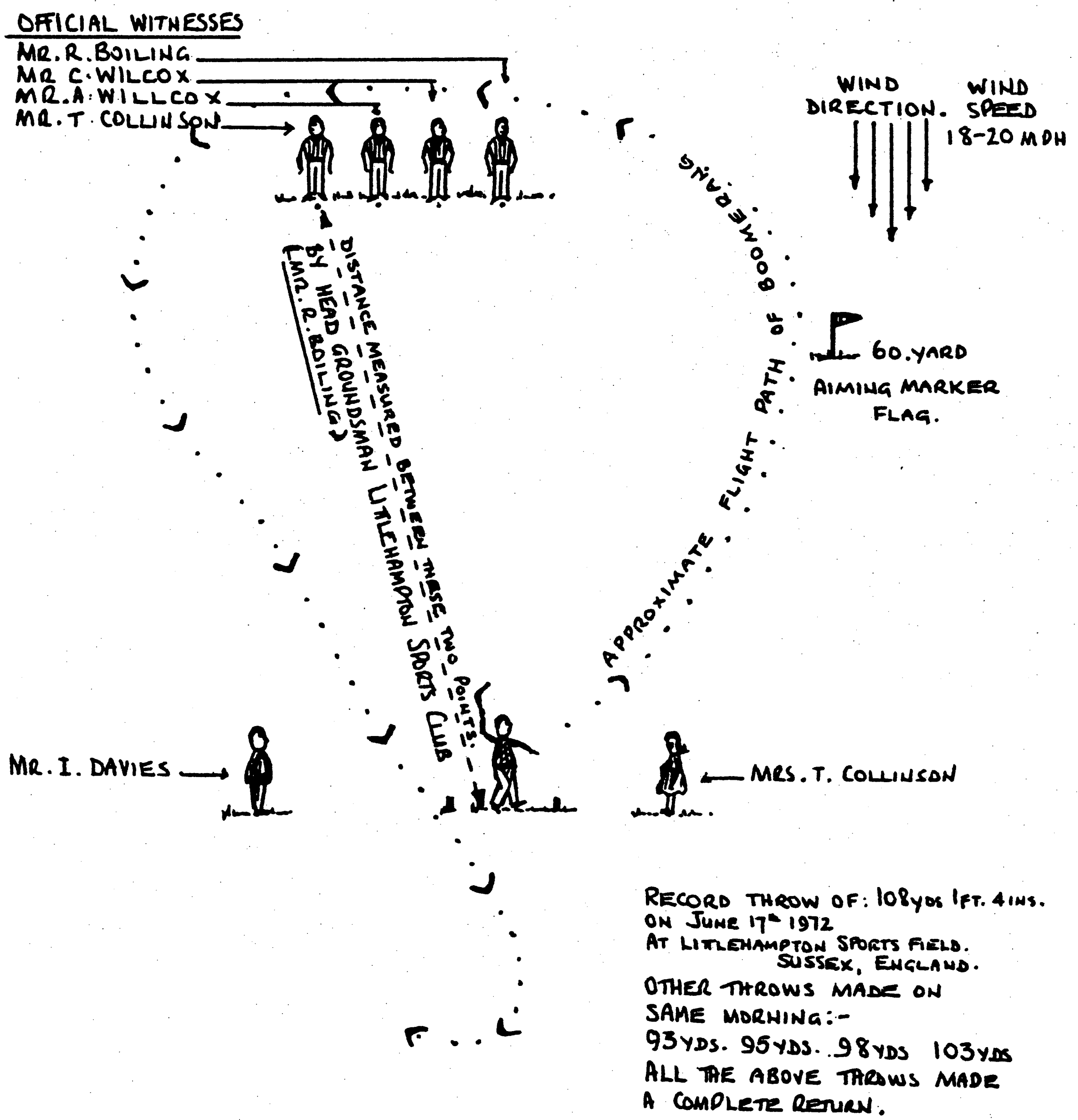


fig. 13.1. Record throw by Herb A. Smith: farthest point: 99.16 m. (Drawing by Mr. Smith.)

A returning boomerang typically reaches a maximum distance of some 30 m from the point of launching, and a maximum height of say, 15 m. Lightweight boomerangs for indoor use may have flight paths with a diameter as small as 3m. On the other hand, specially designed boomerangs may reach much larger distances and still return completely. Distances of over 100 m are mentioned in the literature, but usually these cases are not well documented, and rest on optimistic estimates rather than accurate measurements. The one exception, as far as I know, is that of Mr. Herb Smith of Arundel, Sussex. In 1972 he did some long-distance throwing experiments in which witnesses measured the maximum distance reached by the boomerang, and checked if the boomerang fully returned to the thrower or behind him. The record throw by Mr. Smith was done on June 17th, 1972: 99 metres! The prevailing wind speed was about 8-9 m/s. Figure 13.1 shows a sketch of the record flight path, indicating the positions of the witnesses.

§14 *Throwing.*

Returning boomerangs may have various shapes, but they always consist of two or more arms lying approximately in one plane. An essential feature is the cross section of the arms, which is more convex on one side than on the other. More details can be found in §15.

fig. 14.1. The grip for a right-handed thrower. (Copied from [Ruhe, 1972]). The boomerang may also be gripped at its other end, so that the free end point backwards.

A boomerang is thrown by gripping it at one of its extremities, holding it up with the more convex side towards the thrower's cheek, and hurling it forward in such a way that the boomerang is released with a rapid spin. This is not very difficult to achieve: as regards launching a boomerang resembles not so much a ball as a sling. Its centre of mass is situated at some distance from the part gripped by the thrower. The boomerang *swings* forward and acquires a forward speed and a rotational

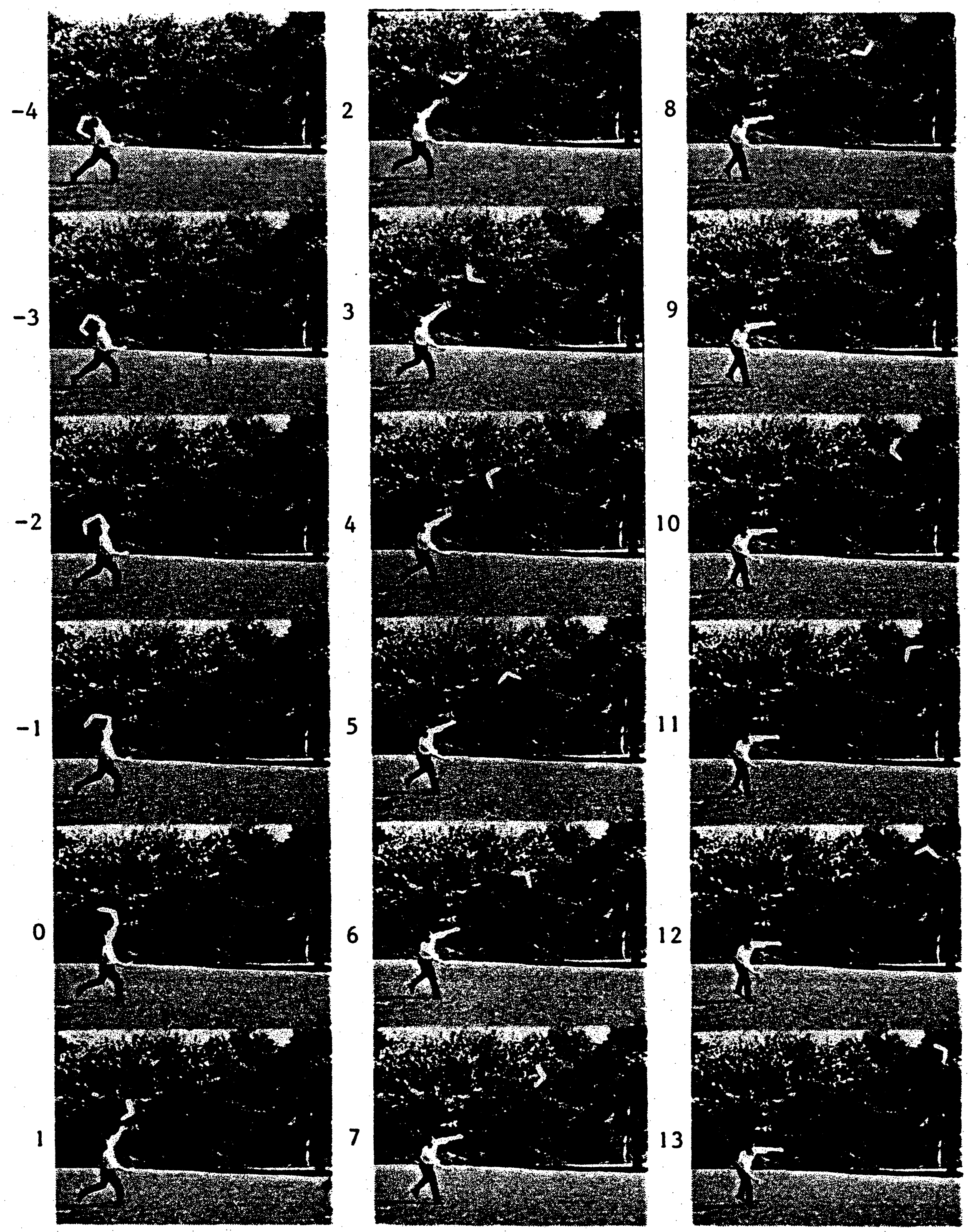


fig. 14.2. Left-handed throw and first part of flight. (16 mm film fragment, taken at 64 fr/s.) Numbers denote the time from the instant of release in units of 1/64 sec.

speed at the same time. At the instant it leaves the hand its centre of mass moves much faster than the thrower's hand itself. In fact it is not at all difficult to launch a boomerang at a speed of 90 km/h. To impart as much spin as possible to the boomerang, take care not to move the hand too fast, try to even stop the hand just before the instant of release. This makes the boomerang pivot about the thrower's wrist, rather than about a lower point such as the thrower's elbow.

Figure 14.2 shows the launching of a boomerang and the first part of the flight. The pictures have been enlarged from a 16mm film exposed at 64 frames per second. Numbers denote time from instant of release in units of 1/64 sec. The left-handed thrower accelerates the boomerang to full speed in 1/10 sec. The boomerang completes one revolution in 6 frames, which indicates an initial spin of 10-11 revs/s. The boomerang's initial forward speed is 25-27 m/s (90-100 km/h).

The angle ϑ between the boomerang's plane of rotation at launch and the horizon (see fig. 14.3) has a profound influence on the flight path.

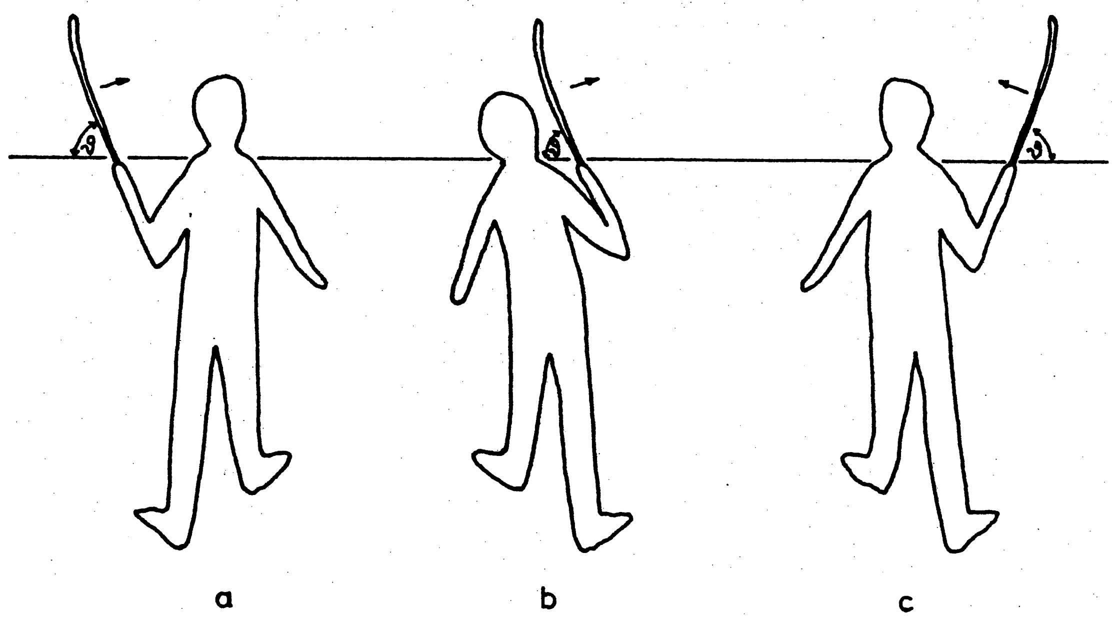


fig. 14.3. View from behind the throwers. ϑ is the angle between boomerang's plane of rotation and horizon. $\vartheta \approx 70^\circ$ in all three cases. Small arrows near the more convex side of boomerangs indicate the direction in which the flight path will curve.
a) left-handed thrower launching left-handed boomerang.
b) right-handed thrower launching left-handed boomerang.
c) right-handed thrower launching right-handed boomerang.

Most boomerangs should be launched at angles ϑ between 45° and 90°. If an ordinary boomerang is thrown at $\vartheta = 0^\circ - 30^\circ$, usually it soars up high in the air, and comes down either fluttering or at a terrific speed. Some boomerangs may behave differently: one of my own, when thrown this way, could traverse a circular loop in a vertical plane: loop the loop.

Usually a boomerang is suited to be thrown either with the right hand or with the left hand. Most boomerangs are *right-handed*: their sense of rotation in flight is counterclockwise as viewed from the more convex side. If such a boomerang is made to rotate in the opposite direction, it generally does not behave like a good boomerang. The mirror-image of a right-handed boomerang, however, should rotate clockwise in order to work well. Such a *left-handed* boomerang is suited to left-handed throwers. In every respect a left-handed throw with a left-handed boomerang is the exact mirror-image of a right-handed throw. The flight path curves to the right instead of to the left, etc. Cannot a left-handed boomerang be thrown with the right hand and conversely? This is indeed possible, see fig. 14.3b. If the boomerang is to be launched at an angle ϑ of about 70° or less, the thrower is forced to assume a rather uncomfortable posture, however. For throws at $\vartheta = 90^\circ$ (plane vertical) the thrower's handedness obviously does not matter.

Good and detailed instructions for throwing boomerangs can be found in the recent booklets: [Ruhe, 1972], [Hanson, 1974] (with special attention to the symmetry between left- and right-handers), [Mason, 1974] (primarily about indoor boomerangs), and, for those who read German, [Urban, 1966] (excellent textbook on the sport of boomerang throwing). Optimum conditions for boomerang throwing are provided by a piece of grass land the size of a football field, without trees or nearby buildings. The weather should be almost windless, although some boomerangs perform best when the wind speed is about 3 m/s. If there is wind, the boomerang should be thrown to the right of windward (for right-handed boomerangs), so that the flight path is traversed almost completely upwind from the thrower. The throw of fig. 13.1 is an example. Always be very careful when people are watching within 50 m distance: boomerangs are capable of inflicting serious wounds.

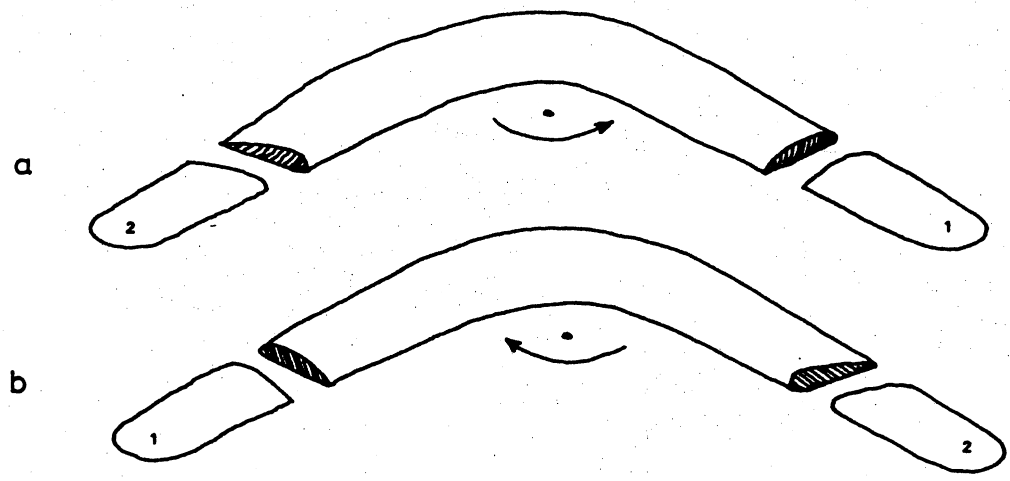


fig. 15.1. a) right-handed and b) left-handed boomerang, each cut at two places to show cross sections. Arrows indicate sense of rotation in flight. Dots indicate centres of mass.

The most important feature of a returning boomerang is the cross section of its arms, see fig. 15.1. This should be more convex on one side than on the other. The detailed shape of a boomerang's planform is less important. The angle included between the arms may vary between 70° and 130°, for boomerangs of the type sketched in fig. 15.1 and 15.2. Quite different planforms are also possible. Returning boomerangs may for instance resemble the capital letters: C, H, L, S, T, U, V, X, Y, Z.

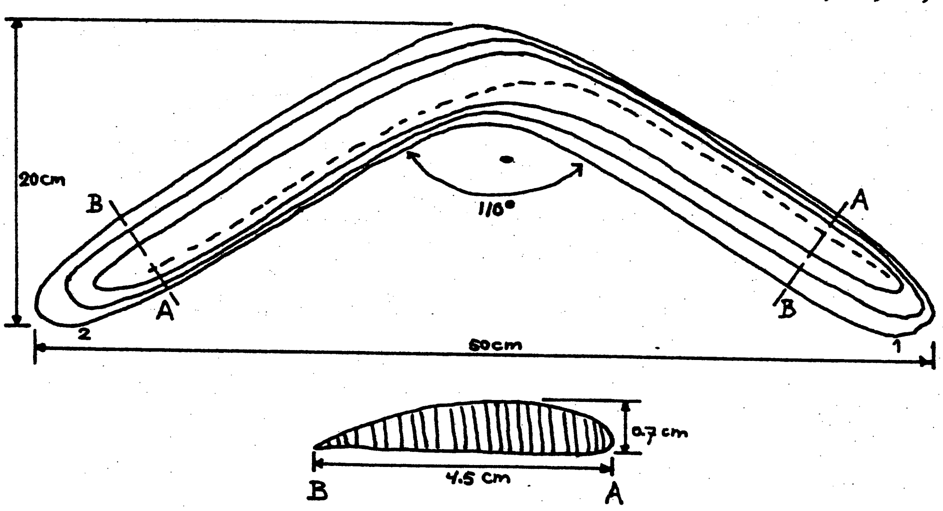


fig. 15.2. Right-handed boomerang, thickness distribution and cross section. A = leading edge, B = trailing edge of arm. —— = lines of constant thickness, ---= thickest part of cross sections.

Making a good returning boomerang is not difficult. A suitable material is plywood of 0.5–0.8 cm thickness. Fig. 15.2 provides a fairly simple example. The dimensions, which are not critical, might be chosen as follows: plywood thickness: 0.7 cm, tip-to-tip length: 50 cm, angle enclosed between arms: 110°, width of arms 4.5 cm, somewhat more at the elbow. The amount of plywood required for one boomerang is 50 cm × 20 cm. The boomerang's weight will be about 130 g. Saw out the planform with a jig-saw. Bring upper side into desired shape with a rasp or a file. The successive plies will be clearly visible, and show whether the obtained shape is smooth and regular. Leave the underside flat. Round the leading edges and the tips, and sand the whole surface smooth. If the boomerang performs well in a couple of trial throws, paint the boomerang with glossy lacquer. Bright colours are convenient when the boomerang occasionally does not return after flying into a tree. For a left-handed boomerang interchange A and B in fig. 15.2. A double-handed (i.e. both left- and right-handed) boomerang may be obtained by carefully giving its cross sections a symmetric shape (see fig. 15.3d). The dimensions given here may be changed by as much as a factor of 2 upwards or downwards.

The cross sections shown so far have a blunt leading edge, a sharp trailing edge and a smooth surface. This may not be necessary. Some boomerangs perform well with rough surfaces or pieces broken off, and some may have profiles as badly shaped as the one shown in fig. 15.3g.

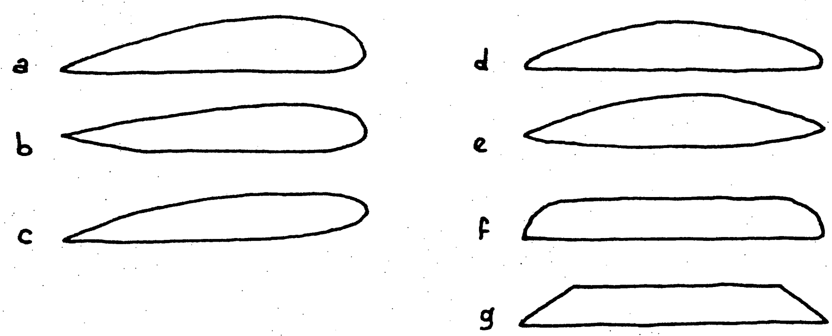


fig. 15.3. Some profiles which might be used for boomerangs. Leading edges at right.

The practice of making and throwing boomerangs suggest the following rules of thumb. If a boomerang "lies down" too much, soars up, and does not return, but describes only an open loop, it may help to increase the "lift" on arm 2 by filing away a bit of the underside near the leading edge, as shown in fig. 15.3c. On the other hand, if the boomerang "lies down" too little, so that after describing half a loop it loses height too fast, file away a bit of arm 1 in a similar manner. A boomerang's hovering qualities may be improved by filing away a bit of the undersides of both arms near the trailing edges, as shown in fig. 15.3b, especially at the tips. If one desires to increase the dimensions of a boomerang's flight path, ballast may be attached near both wing tips, preferably inside the boomerang, or on the flat underside.

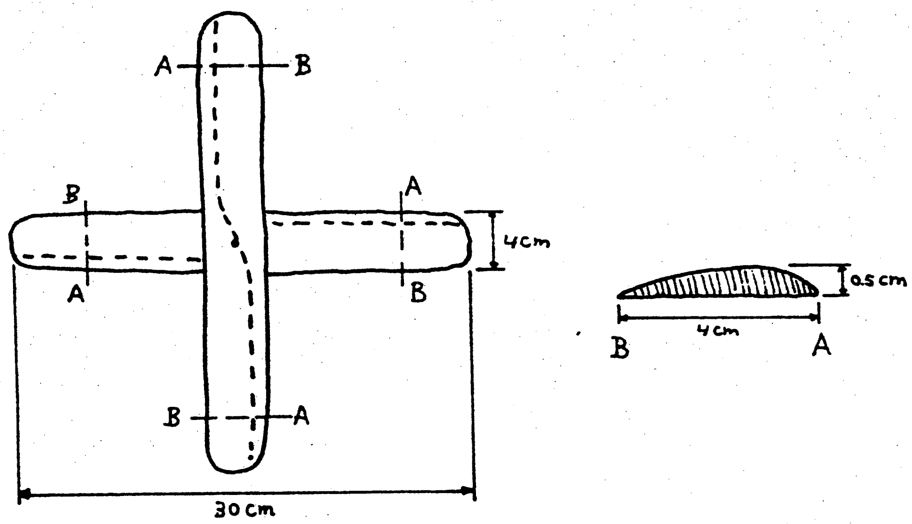


fig. 15.4. Cross boomerang.

A boomerang suitable for indoor use is shown in fig. 15.4. It should be very light: 15-20 g. Use balsa wood of about 0.5 cm thickness. Cut out two laths of 4 cm × 30 cm. Bring the upper sides of the arms into shape with file or knife, and glue one lath upon the other, the upperside of which has been left flat near the centre. This boomerang may fly better with some ballast attached at the centre.

Good instructions for making boomerangs can be found in: [Ruhe, 1972], [Hanson, 1974], and [Mason, 1974] (cross boomerang type). As to materials other than wood (polypropylene, magnesium), one might consult Mr. G. Rayner [1972].

§16 *The principles of the returning boomerang.*

This section presents a rough elementary explanation of the remarkable behaviour of returning boomerangs. First of all, note that if a boomerang would be thrown in a complete vacuum (on the moon for instance), it would traverse a parabola, just like any other object. The gravitational force (= its weight) would pull it downwards, and no other forces would act on it during its flight. It would behave just like an ordinary stick. (If gravity would be absent too, the boomerang, like any other object, would fly in a straight line at a constant speed.) If a thing does *not* fly like a stick or a stone, it is because of forces exerted upon it by the air. Obvious examples of such things are: airplanes, birds, insects, maple seeds, pieces of paper, snow flakes, autumn leaves, boomerangs, etc. A boomerang would be no boomerang, without air to move through.

It is the aerodynamic forces which curve the boomerang's path. However, not every piece of wood is capable of generating these forces the way a boomerang does: it must be properly shaped. Moreover, even a good boomerang, dropped from a window cannot be expected to automatically return, it must be properly launched. Hence two factors play key roles: the boomerang's shape, and the boomerang's motion. More specifically, any explanation of the return behaviour of boomerangs must be based on these two principles:

1. the boomerang's arms are wings,
2. the boomerang spins rapidly and behaves as a top.

Let us first consider principle 1: *the boomerang's arms are wings.* As we remarked earlier, an essential feature of a boomerang is the shape of the cross section of its arms. Usually this cross section is more convex on one side than on the other, see fig. 15.1. In this respect boomerang arms resemble the wings of airplanes and birds. If an airplane flies horizontally through the air, its weight must be counterbalanced by an upward force, or it would fall down. As airplanes are heavy, this upward force must have a considerable magnitude. Where does it come from? Obviously, it comes from the air and acts on the airplane's wings.

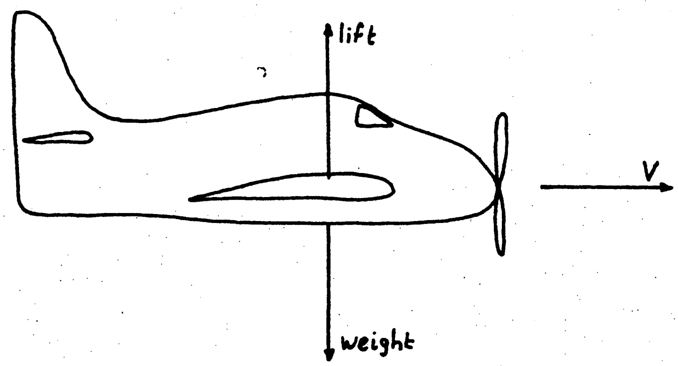


Fig. 16.1. Airplane, flying horizontally at speed V. Weight is counter-balanced by upward aerodynamic force called lift.

If an arbitrary object moves through the air, the air flows around it, giving way to the object. The air, which is originally at rest, is forced into motion, and after the object has passed, it will still move. To bring about the air's motion, the object must exert forces on the air, and the air in turn reacts and exerts opposite forces on the object. In daily life the resulting aeordynamic forces usually are directed opposite to the object's motion: the object, which may be a ball or a bicyclist, is slowed down by the air. However, boomerang arms, like airplane wings, have a special shape, which causes the aerodynamic forces to act in a direction nearly *perpendicular* to the wing's motion. In the case of an airplane the resulting force is directed upwards.

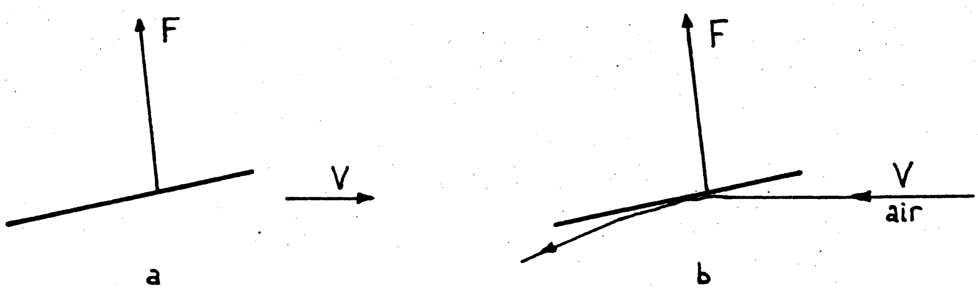


fig. 16.2. a: inclined flat plate moving at velocity V through air.
b: plate stands still, but air moves at opposite velocity V. Both cases are equivalent. F = resulting upward force on plate.

In order to understand the cause of this force, consider a flat, thin plate, which moves horizontally forward at a velocity V. The plate is slightly inclined upward with its leading edge. The air will exert a force F on the plate, more or less as indicated in fig. 16.2a. This appears quite plausible: the air is pushed downwards by the plate, and, trying to resist this sudden motion, hits the underside of the plate, which experiences an upward force F. Perhaps it is easier to understand the eqiuvalent situation in which the plate stands still and the air moves in the opposite direction at velocity V, see fig. 16.2b.

However, this explanation is not satisfactory for wings without apparent inclination, which may also experience upward forces. Such wings always are more convex on the upper side than on the underside. A homely example is provided by the umbrella. If the reader has on accasion protected himself in gusty weather against the rain by means of an umbrella, he may have noticed the upward pull on the umbrella at each wind gust (fig. 16.3). Similarly, but on a larger scale, a gale may pull the

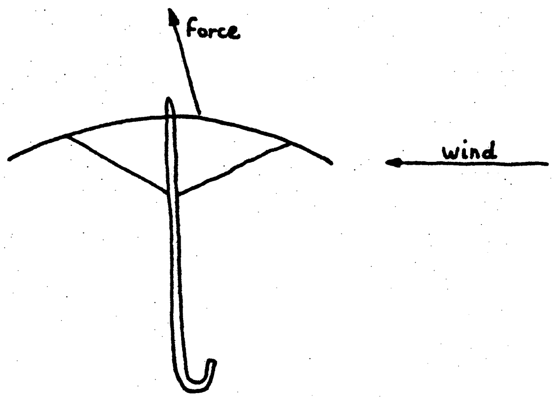


fig. 16.3. Upward force on umbrella held upright in wind.

roof from a house. In these cases the lift originates from the curved path the air is forced to move along, see fig. 16.4. Let us first look

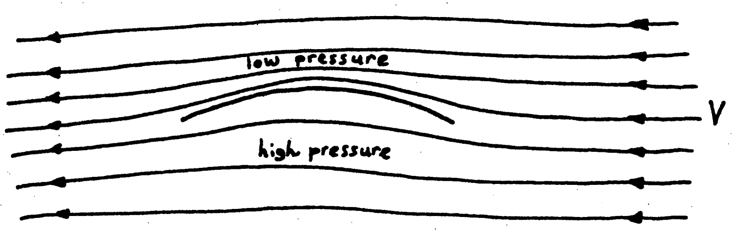


fig. 16.4. Air flow around an umbrella (cross section).

at the convex upper side. The air moves upwards near the rim of the umbrella, which therefore experiences a (small) downward force. But as soon as the air flows over the surface of the umbrella, it must follow its curved shape. The air particles, tending to follow straight paths, are inclined to move away from the umbrella's surface. Indeed, this tendency results in a lowered air pressure over the umbrella (centrifugal effect). Farther away, higher above the umbrella, the air pressure is about normal. It is exactly this pressure difference which prevents the air particles from flying straight away, and makes them follow their curved paths along the umbrella's surface. Now look at the concave underside. Here, too, the air particles must follow curved paths. In tending to fly straight, they push against the umbrella. They must be continually deflected downwards from the direction of their momentary velocity. This increases the air pressure under the umbrella. We see that there is a difference in air pressure on both sides of the umbrella: higher pressure underneath, lower pressure on top. These respectively push and pull the umbrella upwards. If your hand holds the umbrella in a fixed position, you feel an upward pull.

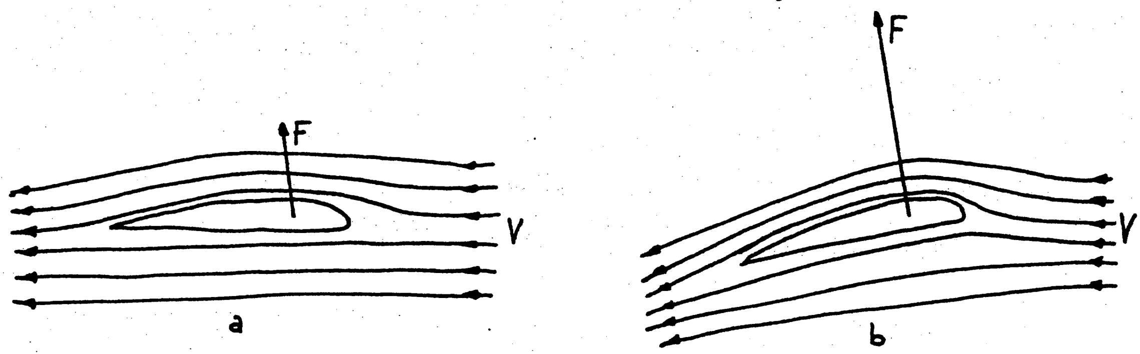


fig. 16.5. Air flow around wing (cross section). F = resulting force. a: no apparent inclination. b: with additional inclination, F is larger.

For airplane wings and boomerang arms, the story is essentially the same. The air pressure on the more convex side is lower than on the other side, and a net force F results, see fig. 16.5a. If the wing is inclined more and more, as indicated in fig. 16.5b, the resulting force becomes larger (up to a certain limit). If the velocity of the air is increased, the force F increases too. Or, for moving wings in still air: the greater the wing speed V, the greater the force F. Actually, if V is doubled, F increases fourfold (within a limited, but wide range of velocities V). Now we know what keeps an airplane up in the air.

As stated in §14, a right-handed returning boomerang is usually thrown in such a way that its plane of rotation is nearly vertical, the more convex side facing towards the left. As the aerodynamic forces exerted upon the boomerang arms are directed from the less convex side to the more convex side, the resulting force, instead of pointing upwards as with an airplane, points towards the left, as seen by the thrower (fig. 16.6). This force accelerates the boomerang leftwards. One might therefore expect the boomerang to swerve to the left, as indeed it does. However, this is only one part of the explanation.

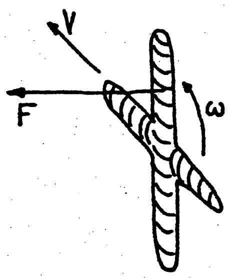


fig. 16.6. Resulting leftward force F on right-handed boomerang.

In the following, we shall refer to a cross boomerang, just for convenience. For differently shaped boomerangs the explanation is the same. The length of the boomerang's arms (from boomerang's centre of mass to tips) is a. The boomerang has a forward speed V, and a rotational speed ω. At each instant, not all parts of the boomerang have the same velocity. This is due to the combination of the forward speed and the rotational

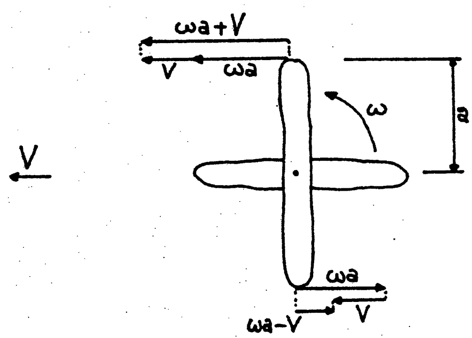


fig. 16.7. Upward pointing end of boomerang has velocity $\omega a + V$ in forward direction, downward pointing end has velocity $\omega a - V$ in backward direction.

speed. The upward pointing end of the boomerang moves faster that the downward pointing end. In the first case (top end) the forward speed V adds to the speed due to rotation ωa; the resulting speed is $\omega a + V$. In the second case (bottom end), these speeds are in opposite directions and must be subtracted; the resulting speed is $\omega a - V$ in backward direction (see fig. 16.7). Compare this with a rolling wheel, moving at a forward speed V: the upper most point has an instantaneous speed 2V, the point touching the ground stands still (here $\omega a = V$). Hence the uppermost

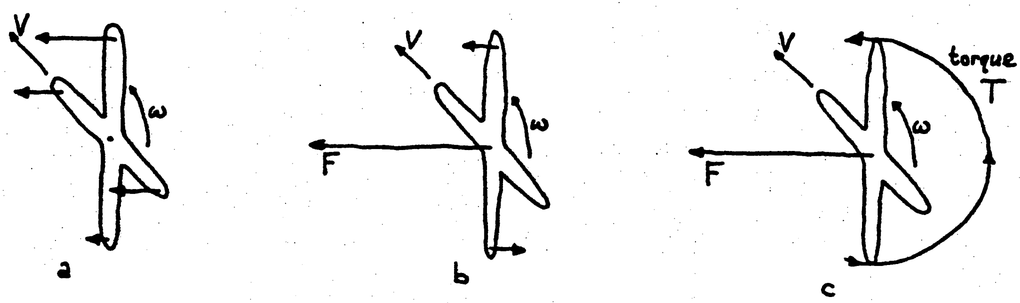


fig. 16.8. a: distribution of leftward forces: stronger at the top, weaker at the bottom. b: this is equivalent with a resulting leftward force F acting on the boomerang's centre, and additional leftward forces at the top and rightward forces at the bottom. c: these form a torque T, which tries to cant the boomerang with its top towards the left.

parts of the boomerang experience much stronger leftward forces than the lower parts do. This means that the aerodynamic forces not only produce a net leftward *force* F, but also a net *torque* T, which tries to cant the boomerang with its upper part to the left: counterclockwise as seen from the thrower. See fig. 16.8. This canting would be about an imaginary horizontal axis, called the torque axis. However, we do not observe such canting in boomerangs!

At this point we must consider principle 2: *the boomerang spins rapidly and behaves as a top.*

Put a top upon its peg, and it will, of course, topple over. But give it a fast spin, and it can stand upright. The difference is due to the rapid rotation. A spinning top reacts in a peculiar way to an applied torque: it does not give way to the torque, but rotates slowly about an imaginary axis perpendicular to both the spin axis and the torque

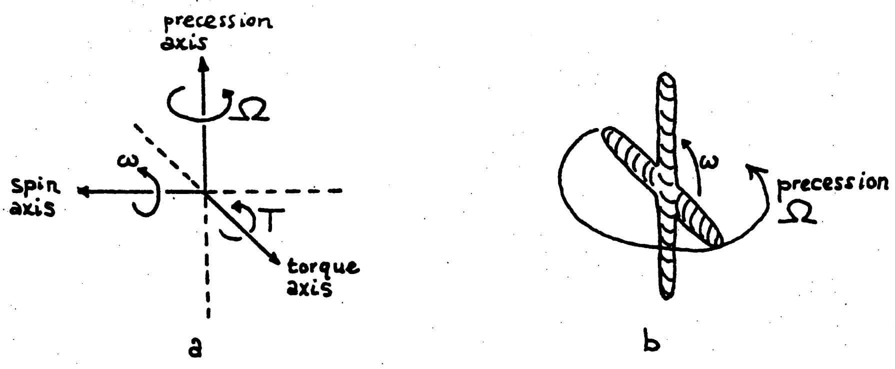


fig. 16.9 a) Precession Ω about axis perpendicular to both torque axis and spin axis. b) Precession of boomerang.

axis. See fig. 16.9. This motion is called *precession*. A boomerang behaves just the same way. Here the spin axis is horizontal, to the left, the axis of the aerodynamic torque T is horizontal, directed backwards towards the thrower, and the axis of precession is vertically upwards: the boomerang moves with its foremost part to the left and rotates slowly with an angular precession velocity Ω counterclockwise as viewed from above. Thus the boomerang turns its foremost part, rather than its uppermost part, to the left. In daily life, this phenomenon of precession is exploited, when one bicycles "with no hands" through a curve: leaning to the left makes the spinning front wheel turn to the left.

A rough explanation of this precession is as follows. Here we shall disregard the net force F, as its only effect is to accelerate the boomerang as a whole to the left. The net torque T then originates from

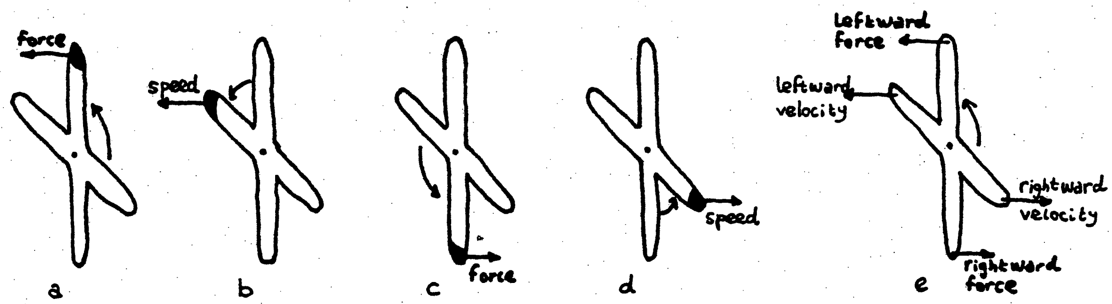


fig. 16.10. Precession of boomerang. a,b,c,d: one wing tip in the course of a revolution. e: front part has leftward velocity, rear part rightward velocity.

leftward forces on the upper part of the boomerang and rightward forces of equal magnitude on the lower part of the boomerang (see fig. 16.8b). The maximum leftward force is exerted on the upper part of the boomerang (fig. 16.10a), this part is accelerated to the left and gradually acquires a leftward speed, which reaches its maximum when this part has become the foremost part (fig. 16.10b). When this part descends further the torque begins to push it to the right, which decreases its leftward speed. The maximum rightward force occurs when the considered part is at its lowest (fig. 16.10c), here the leftward speed has vanished, and a rightward velocity begins to grow, reaching a maximum as the part is half way the top (fig. 16.10d), and vanising when it is uppermost again after having completed one revolution (fig. 16.10a). The result of this sequence is indicated in fig. 16.10e. The combination of leftward velocity in front and rightward velocity behind constitutes the motion of precession: the boomerang slowly rotates its plane about an imaginary vertical axis. The larger the torque T, the faster the precession.

From our explanation so far the following picture emerges. The boomerang originally moves horizontally forwards, its plane of rotation vertical. Soon it swerves to the left because of the net force F. At the same time it responds to the net torque T by slowly moving its foremost part to the left. The combined effects may result in the boomerang's traversing a curved path, and its returning to the point of launching. We shall presently show how. Effects due to gravity (weight) are disregarded. We call Ψ the angle between the boomerang's plane of rotation and the direction of its forward speed. If $\Psi = 0$, the boomerang moves parallel to its own plane. If $\psi > 0$, the boomerang is inclined with respect to its forward motion, and the aerodynamic forces will be larger. This is because each part of the boomerang arms will be inclined too, and experience a larger "lift", see fig. 16.5.

Let us consider the three hypothetical cases indicated schematically in fig. 16.11.

Case a: only the net force F acts, the net torque T is negligible. The boomerang acquires a leftward velocity in addition to its original forward speed. Its plane of rotation remains parallel to itself. This results in the angle of incidence becoming negative: $\Psi < 0$. As Ψ becomes

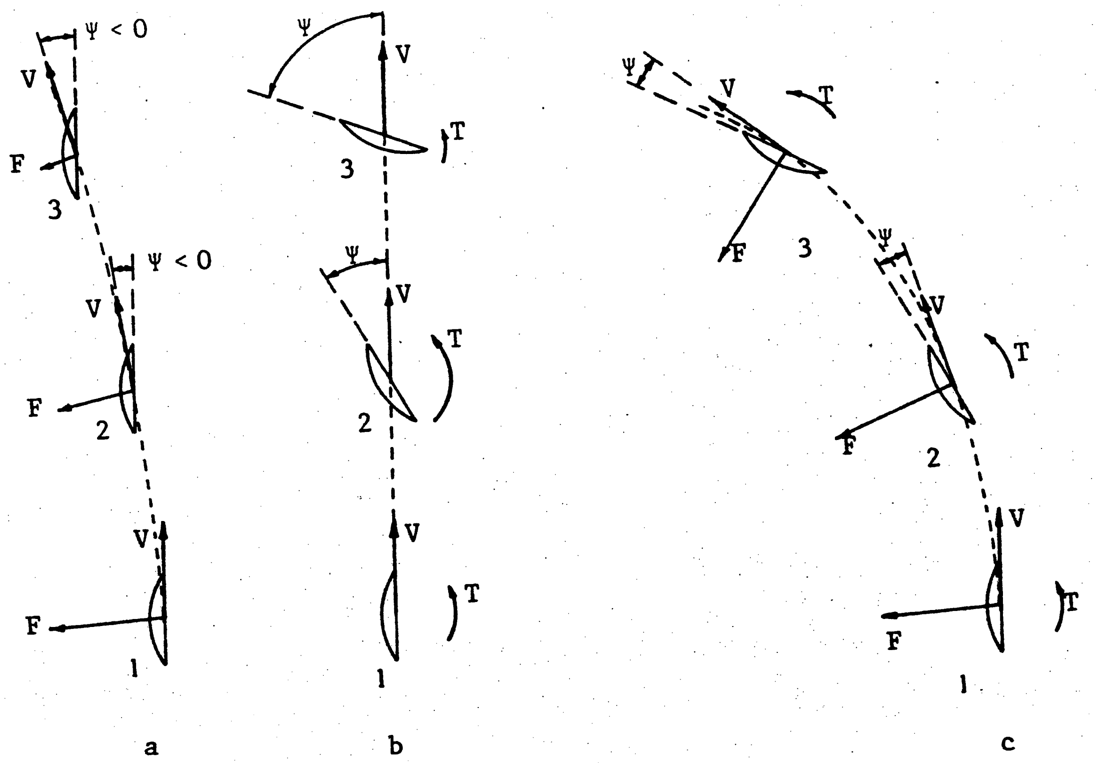


fig. 16.11. a) only force F acts. b) only torque T acts. c) both F and T act. V is forward velocity, Ψ is angle of incidence. 1,2,3: successive positions, bird's-eye view.

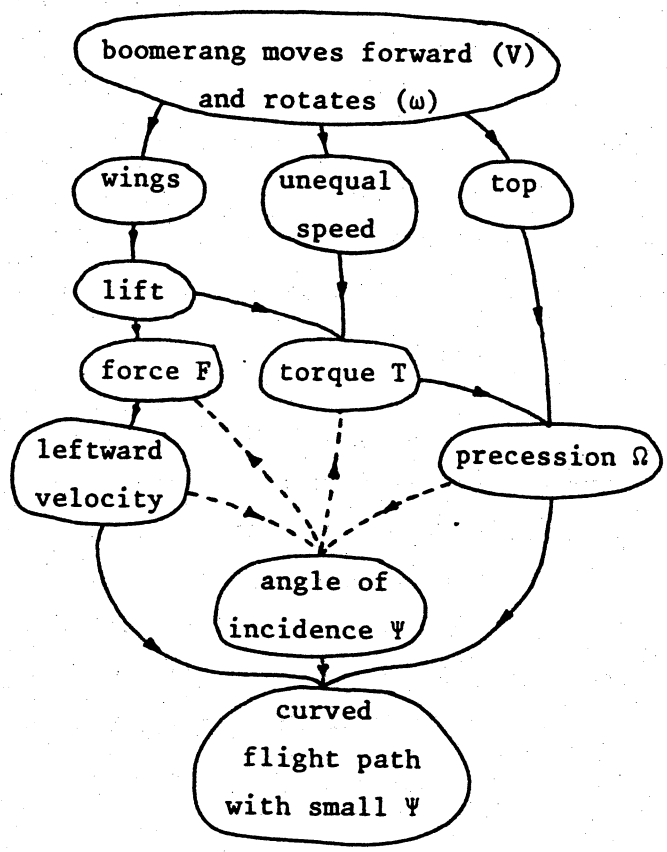
more negative, the force F decreases until it vanishes. The boomerang finally flies in a straight line at a constant, negative Ψ.

Case b: only the net torque T acts, the net force F is negligible. The boomerang flies forwards in a straight line, at a constant speed. Meantime the precession rotates the boomerang counterclockwise as seen from above. This increases the angle of incidence Ψ further and further. If the torque T does not vanish before Ψ has reached 90°, the boomerang will finally move with its plane perpendicular to its flight path.

Case c: both F and T act. With a good boomerang both effects are neatly balanced. If the torque T causes Ψ to increase, F will increase also, pushing the boomerang to the left and keeping Ψ from increasing too much. The result is a curved flight path, traversed at a rather small angle of incidence Ψ.

The above explanation makes it understandable how a boomerang can traverse a more or less circular loop, and return to the thrower. During the flight a boomerang is pulled down by its weight, and it should of course complete its loop before dropping to the ground. If the boomerang moves with its plane not vertical, i.e. if $\vartheta < 90°$, the force F may have an upward component, which counteracts the weight, and keeps the boomerang in the air longer.

The main factors in the explanation given in this section are summarized in the following scheme:



The following aerodynamic observation might assist the reader's understanding of boomerangs; it concerns the relation between a wing's "inclination" and the aerodynamic lift. A wing of any cross section can be moved forward at such an "inclination" or "angle of attack" that the resulting lift is exactly zero (see fig. 16.12a), and only a small backward resistance (drag) is present. For a flat cross section this direction is obviously parallel to the wing's plane. For wings with a more convex upper side this direction corresponds to an apparent negative inclination. At any other angle of attack the wing will develop lift (see fig. 16.12b). The angle (α) between the inclination of zero lift

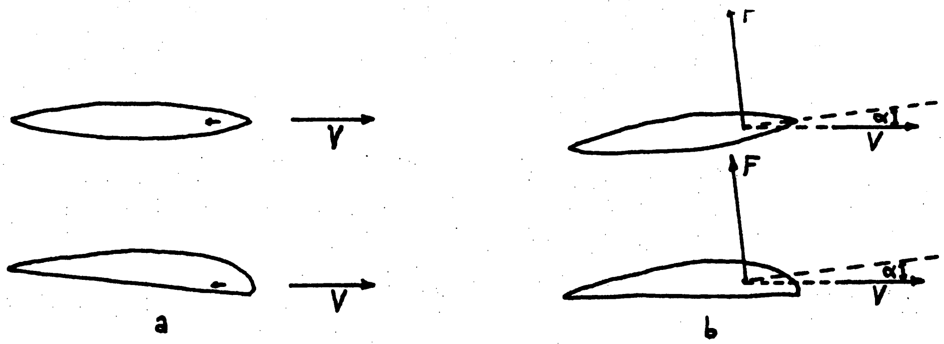


fig. 16.12. Effective angle of attack α for wing sections.
a) Inclination with vanishing.lift (only small resistance), α = 0.
b) With lift. Inclination with respect to direction of zero lift is α.

and the actual inclination is called the section's *effective angle of attack*. If this angle α is not too large (say $|\alpha| \lesssim 10°$), the lift is approximately proportional to α. Thus we see that boomerang arms having symmetrical biconvex or plane cross sections may experience lift, provided the boomerang's angle of incidence Ψ is not zero. Hence, completely flat boomerangs, cut from cardboard, may be capable of performing return flights, although their performance generally is rather poor. If such a plane boomerang (or one with symmetrical biconvex sections) is *twisted*, so that the leading edges of its arms are raised as indicated in fig. 4.1, it will develop lift even at Ψ = 0 (except for the central part which is not inclined). In these cases, the effective angle of attack α of the boomerang arms is not zero, but positive.

§17 *More boomerang mechanics.*

This section deals with five subjects. A: The size of a boomerang's flight path. B: Lying down. C: Straight-flying boomerangs. D: Straight boomerangs. E: Surface roughness.

A: The size of a boomerang's flight path.

Suppose a boomerang flies approximately along a horizontal circle, with its plane of rotation vertical ($\vartheta = 90°$), and with a small, constant angle of incidence Ψ. The boomerang's forward velocity be V (m/s), its spin ω (rad/s), (1 revolution = 2π radians). For a rapidly spinning object, the precessional velocity Ω (rad/s) is related to the torque T and the spin ω according to the formula:

$$\Omega = \frac{T}{I\omega} \tag{17.1}$$

Here I is the object's moment of inertia with respect to the spin axis.

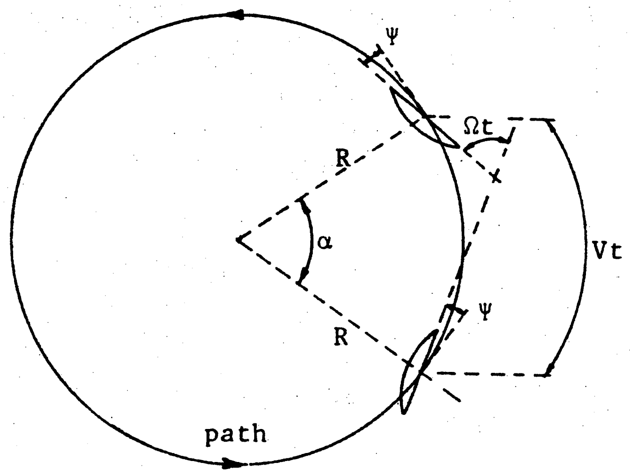


fig. 17.1. Boomerang's position at two instants, t seconds apart. (Bird's-eye view).

Let R be the radius of the circular flight path (see fig. 17.1). In t seconds the boomerang traverses an arc with a length of Vt metres. The angle, as seen from the path's centre, covered by this arc be α, so that the arc's length equals αR. Hence: $\alpha R = Vt$. In the same time interval the boomerang precesses over an angle Ωt. If the boomerang's angle of incidence Ψ (angle between boomerang's plane and flight path) is to be constant, we have the condition $\Omega t = \alpha$. Hence $\Omega t R = Vt$ and

$$\Omega R = V \tag{17.2}$$

To make the boomerang fly along a curved path with radius R, a centripetal (directed towards circle's centre) force is required of magnitude mV^2/R, where m is the boomerang's mass. This force, of course. is supplied by the aerodynamic force F of §16. Therefore:

$$F = \frac{mV^2}{R} \tag{17.3}$$

For the flight path radius R we obtain:

$$R = \frac{mV^2}{F} \tag{17.4}$$

Also, from (17.1) and (17.2) follows:

$$R = \frac{V}{\Omega} = \frac{I\omega V}{T} \tag{17.5}$$

From (17.4) and (17.5) follows the condition:

$$\frac{T}{I\omega} = \frac{F}{mV} \tag{17.6}$$

Both T and F depend on the angle of incidence Ψ. Therefore Ψ must have such a value that (17.6) is satisfied. We emphasize, however, that the way of flying described here needs not always be possible (see Part III, §6).

What happens if one launches the same boomerang at a higher speed, so that both V and ω are increased? Does the flight path become larger? Let us see. According to (17.4) R seems to increase if V does. On the other hand, F also increases with V. If we assume that the ratio ω/V is the same at each launching (which seems not unreasonable), and, moreover, that also Ψ remains the same, then F turns out to be proportional to V^2 (or to ω^2 or to ωV) according to aerodynamic theory. Hence R remains unchanged, according to (17.4). (Alternatively, one might use (17.5) and note that T is proportional to ωV.) This means that the flight path radius is independent of how fast one launches the boomerang! In a sense: each boomerang has its own flight path radius. This is indeed confirmed by experiments.

If a boomerang is made more massive (by making it from heavier material, or by attaching ballast), so that both m and I are increased, but its shape remains the same as before, (17.4) and (17.5) show that the flight path radius increases. The above rough qualitative results were derived as the outcome of a simple boomerang theory in [Hess, 1968a]. A more accurate treatment of this matter is presented in Part III, Ch. I.

B: Lying down.

Generally, the foremost part of a boomerang experiences more "lift", i.e. a larger leftward force, than the rear part does. This is caused mainly by "wake effects". The air, as it passes the boomerang, gradually acquires an induced rightward velocity, because it is pushed to the right by the boomerang's arms. The rear part of the boomerang does not meet "virgin" air, but air moving slightly to the right, so that this part experiences less "lift". This causes the net torque T to have a component which tries to cant the boomerang with its front part to the left. Precession then tilts the boomerang with its uppermost part to the right. The result is visible as "lying down". (In other words, the axis of the torque T is not exactly horizontal, as indicated in fig. 16.9a, but tilted a bit upwards. The precession then proceeds about an axis not exactly vertical, but tilted a bit forwards.)

Another possible cause of lying down is discussed in [Hess, 1968a]. Consider a boomerang of the common type (see fig. 17.2). Both arm 1 and arm 2 experience a maximum leftward "lift" when they point upwards. In

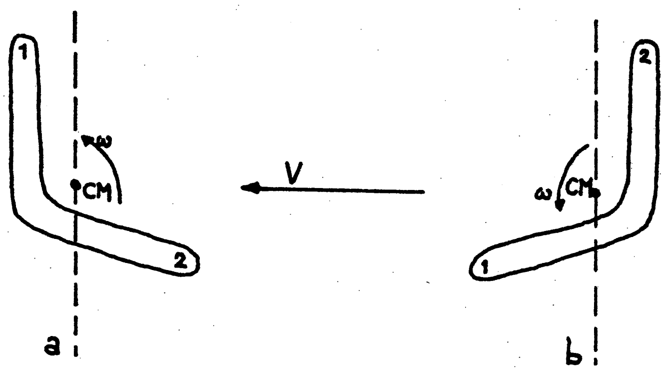


fig. 17.2. a) Maximum "lift" on arm 1 in front of CM. b) Maximum lift on arm 2 behind CM. CM = boomerang's centre of mass.

the case of arm 1 the position of the largest force is in front of the boomerang's centre of mass, in the case of arm 2 it is situated behind this centre. If the "lift" on arm 1 is increased by giving this arm a suitable cross section, or more "inclination", and/or the "lift" on arm 2 is similarly decreased, there results a torque component trying to cant the boomerang with its foremost part to the left, which results in lying down by precession.

In some boomerangs the phenomenon of lying down may be so strong, that somewhere halfway its flight, the boomerang's plane becomes almost horizontal, and, still lowering its advancing part (once its upper part), the boomerang begins to curve to the right. Such a boomerang may describe a path with an 8-shaped planform. The first loop is traversed counterclockwise, the second loop clockwise (see for instance fig. 18.1).

C: Straight-flying boomerangs.

After having read the present chapter up to this point, the reader may wonder whether straight-flying boomerangs are possible at all. They are. Suppose we launch a boomerang in a horizontal direction with its plane approximately horizontal ($\vartheta \approx 0$), and that the net force F, which is a real lift in this case, just balances the boomerang's weight. Suppose further that the net torque T would vanish ($T \approx 0$), then the precession would be absent, and the boomerang would keep its plane horizontal: it would fly straight on. How can we provide a boomerang with a positive net force F and a zero net torque T? Give the boomerang's wings a negative inclination at the tips, and a positive inclination near the boomerang's centre. One might call the result a negative twist. The lift distribution then would have a negative part near the tips, and a positive part in the middle, as indicated in fig. 17.2b. A similar explanation of straight-flying boomerangs was given by Musgrove [1974], who remarks:

> It is important to recognise that the lift and moment distributions required by straight-flying boomerangs [..] are appreciable more complex than those required for a return boomerang. Because of this return boomerangs are easier to construct than straight-flying ones. [Musgrove, 1974, p. 189]

Computed flight paths of a theoretical straight-flying boomerang are shown in Part III, §35.

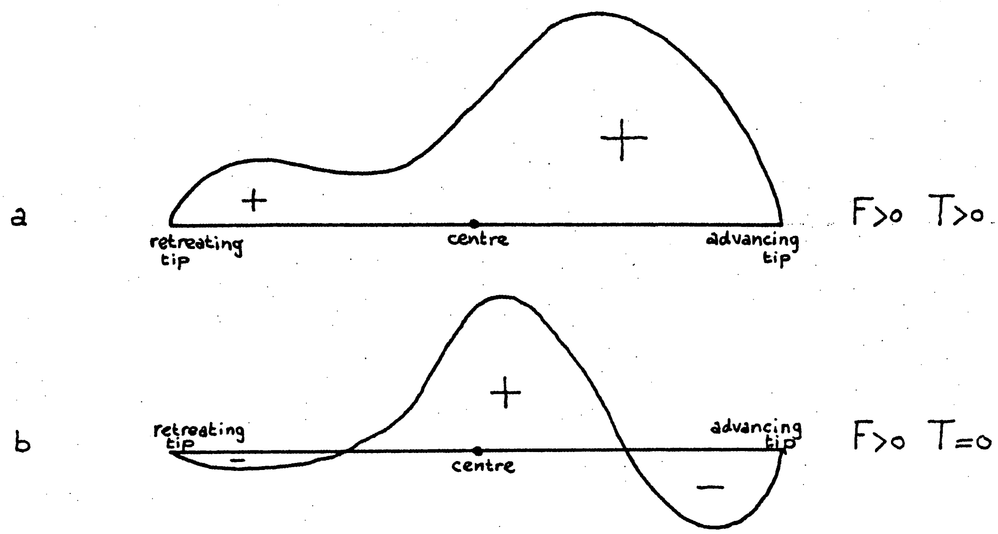


fig. 17.2. a: lift distribution for a returning boomerang, b: lift distribution for a straight-flying boomerang. Views from the (right-handed) thrower's position.

D: Straight boomerangs.

The list in §15 of capital letters which boomerangs may resemble does not contain the I. A completely straight boomerang would not work easily (see §9 under Celebes). Yet everything in §16 seems to be valid for such I-shaped boomerangs too. Indeed it is, provided the boomerang spins the correct way. This is a matter of stability.

According to the theory of rigid bodies: in the absence of external forces and torques, an object can have a stable rotation about either of two axes through its centre of mass, the one with the *greatest* and the one with the *smallest* principal moment of inertia (p.m.i.).

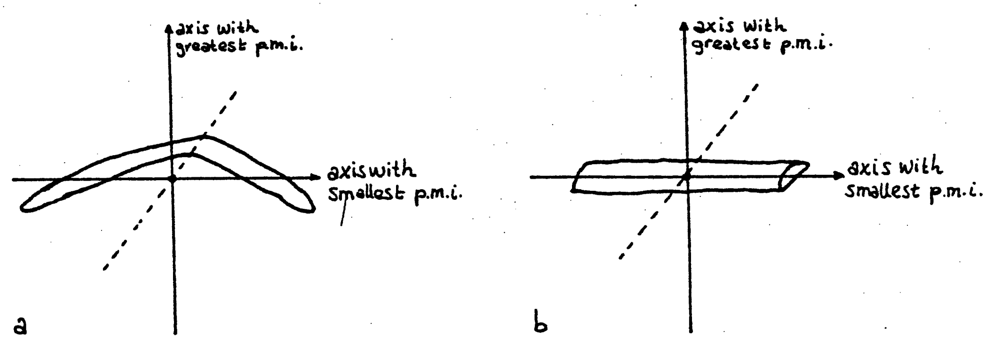


fig. 17.3. Axes about which stable rotation is possible. a: ordinary boomerang, b: straight boomerang.

An ordinary boomerang, which is an approximately plane object, can spin stably about an axis through its centre of mass perpendicular to its plane (greatest p.m.i.), see fig. 17.3a. A straight knitting needle can spin stably about its (longitudinal) geometrical axis (smallest p.m.i.). If the *middle* p.m.i., which belongs to a third principal axis (dashed in fig. 17.3), equals the greatest (resp. the smallest) p.m.i., the rotation (in the absence of torques) about the axis with greatest (resp. smallest) p.m.i. is not stable. The middle p.m.i. of a straight boomerang is only slightly smaller than the greatest p.m.i. (see Part III, §2).

A straight boomerang might be able to spin correctly, i.e. rotate in its own plane about the axis with the greatest p.m.i., see fig. 17.3b. But there are external forces and torques acting on the object, and there is not much to prevent the "boomerang" from starting to rotate about its longitudinal axis (smallest p.m.i.). This undesired rotation probably would be stable, and the boomerang's cross sections would expose themselves in quite unsuitable orientations to the oncoming airflow. The "boomerang's" behaviour would then resemble that of a narrow strip of paper. This longitudinal spin is exploited with a toy called "tumblestick" by Mason [1937], [1974, Ch. V].

E: Surface roughness.

The cross sections of air plane wings generally have a streamlined shape, a round nose and a smooth surface. This helps the air to flow nicely along the wing's curved upper surface and reduces the air resistance. However, there are indications that this smoothness might not be optimal for objects having about the size and the speed of boomerang arms. Golf balls, for instance, are dimpled. Model airplane builders mount wires in front of the wing's leading edges, or may even use wings with sharp leading edges. All this may help to make the air flow near the wing *turbulent*. Curiously, this may reduce the air resistance. More information on this point is given in Part II, §26. The rough surfaces and the rather sharp leading edges of some aboriginal boomerangs might be functional. Only experiments can elucidate this point.

§18 *Earlier research on boomerangs.*

The first publication on boomerangs written from a physical viewpoint is [Moore & McCullagh, 1837]:

> In the present case, therefore, it is clear that the continued swerving from the vertical plane must be ascribed to the action of the air. But to compute accurately the mutual action of the air, and of a body endued, at the same time, with a progressive and a rotatory motion, is a problem far beyond the present powers of science. The problem can only be solved approximately; and however we may simplify it, the calculations are likely to be very troublesome. [Moore & McCullagh, 1837, p. 74/5]

About this time boomerangs had become extremely popular in Dublin:

> Of all the advantages we have derived from our Australian settlements, none seems to have given more universal satisfaction than the introduction of some crooked pieces of wood shaped like a horse's shoe, or the crescent moon; and called boomerang, waumerang, or kilee. Ever since their structure had been fully understood, carpenters appear to have ceased from all other work; the windows of toy shops exhibit little else; walking sticks and umbrellas have gone out of fashion; and even in this rainy season no man carries any thing but a boomerang; nor does this species of madness appear to be abating. [Dubl. Univ. 1838, p. 168]

The quotation is taken from an article in the Dublin University Magazine of February 1838, called "The boomerang, and its vagaries." The anonymous author was the first to give a basically correct explanation of the returning boomerang. In his exposition he made use of a cross-shaped boomerang (the first mention at all of a cross-boomerang, by the way). The crucial role of precession is clearly stated. The lying down of boomerangs follows naturally from his exposition. The only imperfection concerns the explanation of lift on wings, but a better understanding of aerodynamics was perhaps impossible at that early time. Unfortunately this article seems hardly to have been noticed by others interested in boomerangs. It was almost fifty years later that a similar explanation was given by Gerlach [1886] in his excellent paper: "Ableitung gewisser bewegungsformen geworfener Scheiben aus dem Luftwiderstandsgesetze."

In the meantime various attempts were made to clarify the mechanics of boomerangs. Erdmann [1869] gave a correct qualitative explanation. In his theory an important factor is the boomerang's inclined underside (windschiefe Fläche). The aerodynamic lift on wings is conceived of in terms of air particles hitting the wing's underside, not unlike sand-grains. (Compare our exposition of the inclined plane in §16.) Such considerations easily lead one into believing that without twist a boomerang cannot return (see also §4). [Erdmann, 1869] contains good sketches of boomerang flight paths, the first of such quality to be published. Stille [1872] attempted to make quantitative calculations of the boomerang's motion. Again, the basis is provided by the "windschiefe Fläche". He tried to obtain the aerodynamic forces and torques on a boomerang by integrating the force acting on each of the boomerang's surface elements. He was not really succesful.

After [Gerlach, 1886], another article of interest is "Fact and fallacy in the boomerang problem" by Emerson [1893]. He speaks in a rather ironical vein of the mathematical approach:

..... the somewhat astonishing literature of the boomerang bristles with the pointed persistance of the one idea, that this dynamic mystery is a case for mathematical formulae, if there ever was one. [Emerson, 1893, p. 78]

Emerson's paper, based on sound skepticism and common sense, does not give a satisfactory explanation of the boomerang's behaviour. It reviews the previous literature on boomerangs, and, for the first time, reports the use of a small boomerang throwing machine.

In 1897 Walker's [1897a] classic "On boomerangs" was published. Walker understood the mechanics of boomerangs very well. He distinguishes "twisting" and "rounding" of the boomerang arms. Like Stille [1872], he tries to integrate the forces on each surface element. He smoothes his equations (compare the approach of [Hess, 1968a] and Part III, §2) and gives purported conditions for stability. Walker also explains the straight-flying boomerang. However, his lengthy mathematical computations get entangled, and do not seem to make much sense. His drawings of observed flight paths are excellent, and have been often copied by others (see fig. 18.1). Walker's other papers [1900], [1901a,b,c] contain nearly the same subject matter, but without most of the mathematics.

Buchner [1905] also published good drawings of flight paths. His approach is mainly experimental. Lanchester's [1908] appendix on boomerangs gives a good explanation of the return flight. He describes three-armed boomerangs. The phenomenon of autorotation (increase of boomerang's spin) is mentioned, and the influence of the wind on the boomerang's flight is discussed. Buchner's later publications [1916], [1918] contain a description of a small boomerang throwing machine.

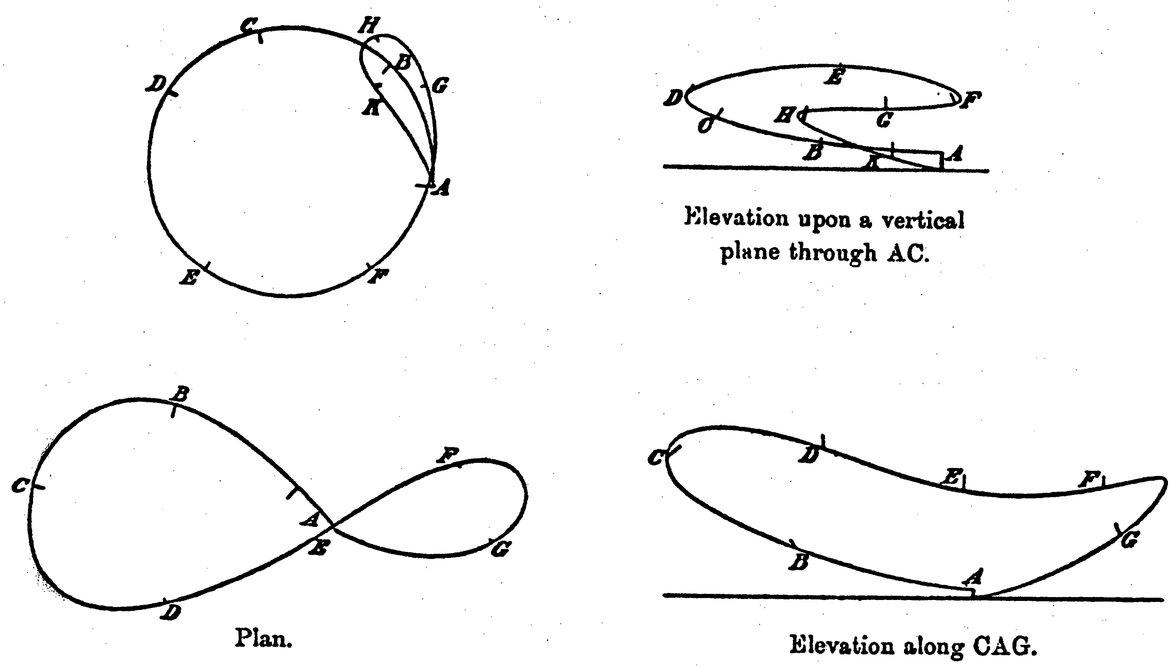


fig. 18.1. Two boomerang flight paths, copied from [Walker, 1897a, p. 39]. Short, straight line segments indicate boomerang's spin axis.

From this time on nothing of significance was added to the scientific knowledge of boomerangs until the late 1960's, with the exception of [Cornish, 1956]. (Recent research is discussed in §19.) When one looks through the literature on the mechanics of boomerangs, one can hardly escape noticing the lack of correlation between the date of publication and the level of the author's understanding of boomerang mechanics. The publications discussed above all are definitively above the average level. But even as late as 1931 a serious physicist asserted that a boomerang is only a spinning discus [Pohl, 1931]. And [Grimsehl, 1923] gives a fully wrong "explanation" of the return flight. These errors are perpetuated in relatively recent physics textbooks: [Pohl, 1959] and [Grimsehl, 1957] respectively. Sometimes a boomerang theory, though erroneous, may at least be amusing; [Landois, 1885], for instance, explains the returning boomerang by analogy with the screw back of a billiard ball. The sparse French literature on boomerangs seems to have followed a separate path. Certainly, the exchange of ideas between English and German authors has been much greater than between either of these and French authors.

Here follows a chronological list of publications, the authors of which contributed, or tried to contribute, to the understanding of boomerang mechanics. With each entry the language of publication is mentioned; E = English, F = French, G = German.
[Moore & McCullagh 1837] E, [Dubl. Univ., 1838] E, [Carroll & Lloyd, 1838] E, [D., 1838] E, [Poggendorff, 1838] G, [Wolff, 1852] G, [Snell, 1855] E, [Lovering, 1858] E, [Erdmann, 1869] G, [Tridon, 1871] F, [Marey, 1871] F, [Stille, 1872] G, [Eddy, 1881] E, [Landois, 1885] G, [Fuchs, 1886] G, [Gerlach, 1886] G, [Eggers, 1888] E, [Emerson, 1893] E, [Walker, 1897a] E, [Walker, 1900] G, [Walker, 1901a,b,c] GEE, [Salet, 1903] F, [Routh, 1905] E, [Sharpe, 1905] E, [Buchner, 1905] G, [Lanchester, 1908] E, [Sutton, 1912] E, [Buchner, 1916] G, [Gray, 1918] E (based on Walker, 1897a), [Buchner, 1918] G, [Grimsehl, 1923] G, [Kreichgauer, 1924] G, [Schuler, 1929] G, [Pohl, 1931] G, [Mottez, 1933] F, [Oxley, 1939] E, [Turck, 1952] F, [Cornish, 1956] E, [Chikazumi, 1967] Japanese, [Mpeye, 1968] E, [Hess, 1968a] E, [Mpeye, 1969] F, [Ruhe, 1970a, 1971a, 1972] E, [Magnus, 1971] G (based on Hess, 1968a), [James, 1971] E, [Hess, 1972a] Dutch, [Rayner, 1972] E, [L., 1972] F (based on Hess, 1968a), [Hess, 1972b] E, [Hess, 1972c] Dutch, [Barger & Olsson, 1973] E, [Hess, 1973a] E, [Jeffrey, Grantham & Hersey, 1973] E, [Musgrove, 1974] E, [Allen, 1975] E, [Hess, 1975] E (this work).

§19 *Recent research on boomerangs.*

The first theoretically calculated flight paths were published in [Hess, 1968a]. On the basis of a simple mathematical model for the motion of boomerangs, equations of motion were derived which could be numerically integrated on a computer. Two main assumptions of this model are: $1^{\underline{o}}$ the air's induced motion is negligible, $2^{\underline{o}}$ the boomerang's angle of incidence Ψ is nearly zero. The resulting theoretical flight paths look surprisingly realistic. They are compared with actual boomerang flight paths, which were recorded as follows. A small light, fed by batteries, was mounted in a boomerang, and the light's trace was photographed at night. Examples are shown in fig. 19.1 and 19.2.

The attachment of light sources to boomerangs has been mentioned much earlier by Mason [1937, p. 34/5] = [1974, p. 34/5]. Cornish [1956, p. 242] used a "thermite flare" to produce a trace bright enough to be photographed: the first publication of a photographically recorded flight path. (In addition, this article contains a correct and clear explanation of the returning boomerang.) [Moulder, 1962, p. 59] also presents a flight path picture by Cornish. Hawes' [no date] commercial boomerang leaflet shows a beautiful photograph of a flight path traversed by a boomerang with a "fourth-of-July-sparkler" attached near its centre of mass. Musgrove and his students use electric lights in their boomerang experiments [Jeffery, Grantham & Hersey, 1973]. In Part III of the present work a number of similarly recorded flight path stereograms are presented. Curiously, also the Australian Aborigines threw illuminated boomerangs at night, as shown by the quotation at the end of §7 from [McConnel, 1935, p. 49/50].

Let us resume our survey of recent boomerang research. Mpeye [1968], [1969] independently did theoretical work, more or less similar in approach to that of [Stille, 1872] and [Walker, 1897a]. Musgrove and his students in Reading (U.K.) are doing theoretical and experimental work on boomerangs. They developed a simple boomerang launcher for use in the open air, to control the initial conditions of the boomerang's flight [Jeffery, Grantham & Hersey, 1973], [Musgrove, 1974]. [Barger & Olsson, 1973] contains an exposition of a simple boomerang theory.

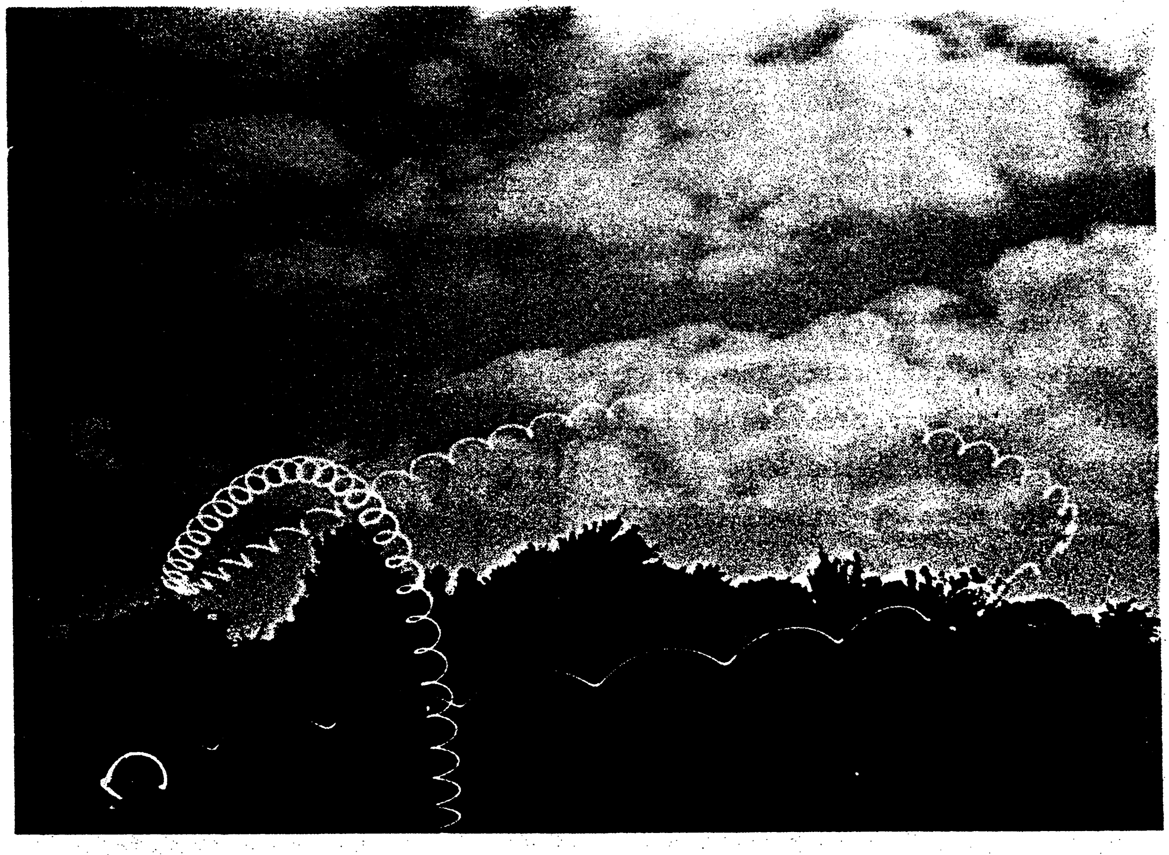


fig. 19.1 (photograph B6, Oct. 1967). Time exposure of the path traversed by a light bulb mounted in a boomerang near one of its tips. Start at lower left, initial direction of the flight to the right, then away from the camera, to the left, towards the camera, and, finally, downwards. Thick part of light trace was made while the boomerang was held by the thrower's hand. Background shows moonlit sky over Groningen Stadspark.

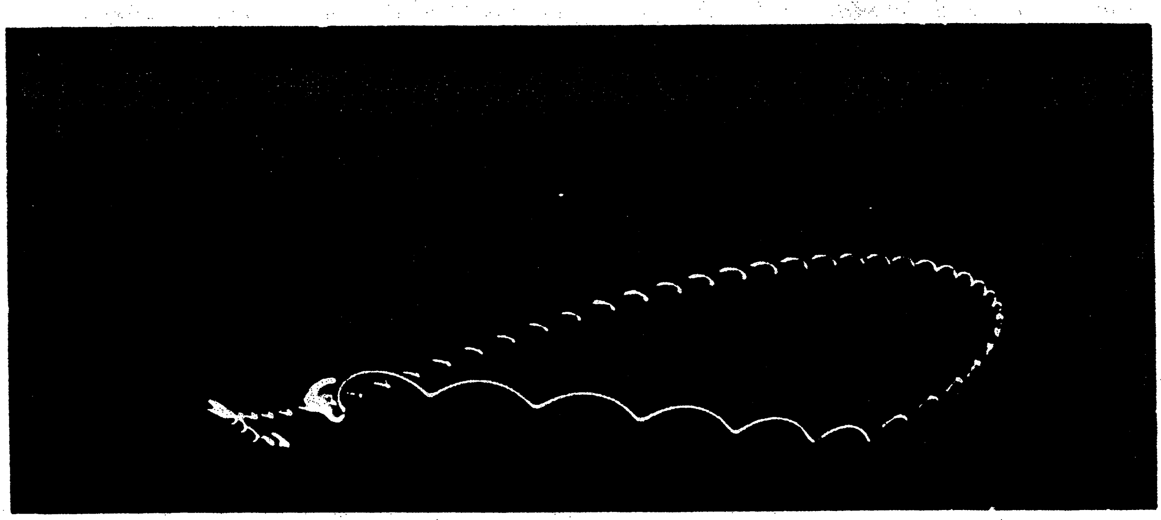


fig. 19.2 (photograph C4, Oct. 1967). Similar time exposure. Here the boomerang hits the ground after flying some 10 metres, which severely diminishes its forward speed.

Allen [1975] and his students did a project on boomerangs. In addition to this research at universities, James [1971] and Rayner [1972] did experimental work on boomerangs, and Ruhe [1970a,1971a,1972] created a boomerang workshop in Washington D.C.

The theory of boomerang motion outlined in Part II and Part III of the present work differs from earlier research on boomerangs in one significant aspect. The induced motion of the air, caused by the forces exerted upon it by the boomerang, is taken into account. Although the theoretical model employed is strongly simplified with respect to reality, it is considerably more complicated than the model of [Hess, 1968a]. To a certain extent such complexity cannot be avoided: The boomerang's motion depends on the forces experienced by the boomerang. These forces depend on the boomerang's motion and on the air's induced motion. This induced motion in turn depends on the forces the boomerang exerts on the air. Both the boomerang's and the air's motion are unknown, and so are the forces exerted mutually by the boomerang and the air. Exactly the same complexity pertains to helicopter rotors, which, in some respects, closely resemble boomerangs. For a description of the main features of our model, see Part II, §1.
Novel are the experiments (reported in Part II, Ch. VI) in which the aerodynamic forces and torques are actually *measured*.

Future research on boomerangs should include, I think, not only the development of theoretical models and experiments better than those reported in this work, but also systematic investigations into (A) the relation between a boomerang's cross section and its performance, and (B) the detailed behaviour of the airflow around rotating boomerangs. The investigations of type (A) would require the making of boomerangs with very precisely determined shapes. Those of type (B) could not be carried out without the use of a wind tunnel and advanced experimental equipment.

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CHAPTER III

BIBLIOGRAPHY.

§20 *Literature on boomerangs.*

Much has been written about boomerangs, and from very diverse points of view. The publications vary widely in content and quality, which reflects the remarkable interest boomerangs seem to arouse in all kinds of people: ethnographers, mathematicians, hobbyists, etc. Curiously, the quality of the information contained in the existing literature hardly seems to be correlated with the year of publication. Since 1836, articles on boomerangs have appeared in all sorts of periodicals, and a lot of them apparently did not come to the attention of later writers, who therefore not seldom started anew or perpetuated old errors.

The bibliography in the next section is a fairly complete one as regards the more important publications. Certainly not everything written about boomerangs is listed here; but each item which I have, at least superficially, read myself is included, with the exception of articles in daily newpapers and cartoons. Some of this literature is rather obscure and difficult to come by. It is not easy to draw a line separating the items to include from those better not included, which is why I included as much as possible. Undoubtedly there exist quite a few articles, unknown to me, which are as least as relevant as many of those listed. If the reader knows about anything written on boomerangs which is not mentioned in the bibliography (or notices errors), I would appreciate his or her informing me about the relevant data (if possible, with a photostatic copy). In the process of assembling the bibliography, which contains some 400 items, [Greenway, 1963] and [Austr. I.A.S, 1973?] were useful sources. Part of the recent literature was brought to my attention by Mr. B. Ruhe.

Since the subject matter of the articles is so diverse, it seemed useful to indicate the nature of each item in the bibliography. This has been done by means of the symbols:

P C A S D B T E F M L R

The main classification is characterized by P, C, A or S:
P denotes: physics, mathematics, science.
C denotes: cultural anthropology, ethnography.
A denotes: archaeology, prehistory.
S denotes: sport.
Most of the articles are written either from an ethnographical/cultural anthropological point of view (C), or from a physical/ mathematical point of view (P). S is mostly used in cases where P cannot be applied very well. Obviously, C and A cannot always be sharply distinguished. C does not distinguish between Australian and non-Australian boomerangs, but usually the title of the publication provides such information. The nature of the content is further indicated by:
D: description of boomerangs, boomerang throwing, or boomerang flights.
B: good pictures of boomerangs (C,A).
T: theory, aerodynamics, mechanics, mathematics (P).
E: experiments (P).
F: pictures of flight paths (C,P).
M: directions for making boomerangs.
L: directions for launching boomerangs.
R: references.
In some instances small letters are used instead of the above capital ones, to indicate the minor relevance to boomerangs, either because of the subject (e.g. African throwing-irons, Melanesian clubs) or because the article adds next to nothing to the earlier literature as regards the considered aspect. Occasional items have a capital letter underlined to indicate an unusual amount of information.

This classification was made for my own use and probably reflects my special interests. A cultural anthropologist, for instance, might prefer a different classification. The 12 classification symbols should be considered as rough indications, rather than as a rigid system. Some arbitrariness was hard to avoid here.

Each item is listed in the bibliography in alphabetical order according to author's surname. The data given are the following. Author's surname, initials, year of publication or submission of article, classification (see above). Further: title of publication, name of periodical, number of volume or issue (underlined), date of publication, page numbers. If only a part of the publication is relevant to boomerangs, the corresponding page numbers are added between square brackets. If a second date is mentioned, it is that of the lecture or of the article's submission. Here the numbers denote: day, month, year, in this order. With books the place of publication is added, though mostly not the publisher's name. Capital letters have been avoided in the titles, except for proper names and German. Author's first names and titles (Dr., Lieut.-Gen., Pater, Lord, Sir) are omitted. With double names no system has been followed, e.g.: A. Lane Fox is listed under Lane Fox with a cross-reference under Fox, R. Brough Smyth is listed under Smyth with a cross-reference under Brough Smyth. A.C. van der Leeden is listed under Van der Leeden with a cross-reference under Leeden. If a publication has more than one author, the item is listed under the first author's name with cross-references under the other authors' names. If an item is part of a work published under a name (B) other than the author's (A), the item is listed under A with a cross-reference under B. Articles written by anonymi are listed either under the inferred author's name, or under the name of the person directly concerned, or else under the (abbreviated) name of the periodical, etc., in each case with the addition: anonymous. Cross-references are listed at the end of the bibliography under Anonymous with years of publication.

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PART II
AERODYNAMICS

Playthings it is indeed hardly correct to call them, as the flight of a boomerang is a scientific puzzle that is never likely to be solved, though many scientists have presented us with learned though usually divergent solutions.

[Payne-Gallwey, 1906, p. 591]

§1. *Introduction*

The flight of boomerangs is a complicated phenomenon: on the one hand, the boomerang's motion depends on the forces exerted by the air, on the other hand, these forces depend on the boomerang's very motion. In its general form a problem like this can hardly be solved. Therefore we have split it in two. Part II of this work deals with the forces exerted on boomerangs which artificially move in a simple way, namely in a constant direction, at a constant forward speed V, a constant rotational velocity ω and a constant angle of incidence ψ. To be sure, in reality boomerangs do not behave like this, but in Part II both the theoretical model boomerangs and the experimental boomerangs are artificially confined to this type of motion. Part III of this work deals with the motion of boomerangs under the influence of forces, which are at first left unspecified. Later we insert rightly or wrongly, the theoretical or experimental forces of Part II into our equations of motion and compute theoretical boomerang flight paths.

Even when moving in the simple way (constant V, ω and ψ) mentioned above, a boomerang experiences forces from the air which are difficult to calculate. As remarked in Part I, §19, the exchange of forces between the boomerang and the surrounding air brings the air into motion. If the air's induced motion would be known exactly, the forces on the boomerang might be computed easily. However, though the boomerang moves in a specified known manner, the air's motion is not known beforehand, as it depends on the unknown forces. One cannot begin to solve this problem unless one makes some definite assumptions, which in our theory concern: A) the influence of the air's motion on the forces experienced by the boomerang, and B) the behaviour of the air when acted upon by forces. The assumptions of kind A made by us are stated in §2, and our assumptions of kind B in §3.

It should be clearly understood that, although such assumptions may be plausible or "not unreasonable" and partly similar to assumptions often made in aerodynamic theories, they may nevertheless lead to invalid results. Some assumptions (e.g. linearity of the air's equations of motion) are known to be of limited validity, but are made because they permit an enormous reduction of the problem's complexity. An assumption like that of the medium's incompressibility may be realistic enough, but some of the other assumptions may disagree with the physical reality. Here it is difficult to be certain: the airflow around a rotating boomerang may have some features which have not been very well investigated at present.(This question is discussed in §26, §32 and §33.) In Part II (with the exception perhaps of §26 and §33) the more physical aspects of boomerang aerodynamics are not considered. Assumptions of kind A and kind B are made, and the problem is forthwith reduced to one in applied mathematics.

Here follows a rough outline of our aerodynamic model. If one traces with a camera the centre of mass of a spinning boomerang in its flight, and takes a time exposure, the result would be a blurred picture. The boomerang arms are smeared out over a circular region. All the positions successively occupied by the arms are shown superimposed. To this picture our so-called *winglet model* is analogous. The boomerang is replaced by a continuous distribution of hypothetical winglets, smeared out over a circular region. The result is a pervious plane structure. The forces exerted by the winglet structure on the air cause the air to move. We suppose that the air behaves like a certain type of idealized fluid. Its induced velocity can then be mathematically expressed in terms of the - as yet unknown - forces. The opposite forces exerted by the air on the winglets can be expressed in terms of the winglets' shape, density and motion and the air's induced velocity. Thus we obtain two relationships between the unknown forces and the unknown velocity field. Together these constitute a rather nasty integral equation, which, after extensive treatment and with the help of a computer, can be made to yield an approximate solution for the forces acting on the winglet structure. It is the "extensive treatment" which, presented in a condensed form, makes up the bulk of the Chapters I, II and III. Since this model resembles a time exposure of a boomerang rather than a boomerang itself, the computed forces can at best be considered as approximations to the time-averages over one period of rotation of the aerodynamic forces on boomerangs.

This winglet model has a curious property: each winglet structure can be simulated by a non-rotating, rigid, porous wing having the same planform (see §4). Actually, our winglet model is a *steady, linearized, porous lifting surface theory*. It contains some features which appear to be novel to lifting surface theory. As the circular lifting surface in our case is pervious, the singularity of the lift function at the leading edge differs from that of a normal wing. The accuracy of the numerical integrations in the computation of the "elementary induced velocities" is independent of the number of pivotal points. However, our approach is ultimately based on Multhopp's method, which is now a quarter century old.

Although the "smearing out" of the original boomerang arms to winglets inevitably brings about a loss of reality, it has the virtue of reducing the original time-dependent problem to a problem of steady flow, which can be tackled much easier. Possibly, models of a different kind would be better suited for application to boomerangs. For instance, an unsteady linearized lifting line theory, in which the boomerang arms are replaced by hypothetical lifting lines, might yield results more realistic than ours.

The Chapters I through V of Part II are mainly theoretical, whereas Chapter VI is primarily a report of experiments. In Chapters I, II and III the winglet model is developed. Chapter IV presents a discussion of the accuracy of the numerical results. The theory is tested against experimental data obtained from experiments with boomerangs in water. In Chapter V the winglet model is modified and extended to make it better adapted to the conditions pertaining to real boomerangs. The most significant feature of the modified *semi-linearized* winglet model is that it can accomodate boomerang arms with arbitrary, non-linear profile lift and drag characteristics.

Chapter VI deals almost completely with experiments on rotating boomerangs in a wind tunnel. The six force and torque components, averaged over time, were measured for five different boomerangs at various combinations of speed V, spin ω, and angle of incidence ψ. In §31, §32 and §33 the measurements are compared with the forces computed on the basis of the semi-linearized winglet model.

Many of the sections of Part II are very technical, and will probably be read only by an occasional aerodynamicist with a special interest in lifting surface theory. He may wish to skip the experimental sections and concentrate on the Chapters I, II, III and V. The reader who wants to follow the main line of Part II is advised to read only the following sections, which include the experiments:

Chapter I: §§2, 3, 4, 5, 7.
Chapter II: §§8, 9.
Chapter IV: §§16, 17, 19.
Chapter V: §§20, 21, 22, 23.
Chapter VI: §§26, 27, 31, 32, 33.

Those readers who wish to learn something about the mechanics of boomerangs but do not like mathematics may want to thumb through the pages, look at the graphs, and start with Part III.

A list of references is given at the end of Part II.

LINEARIZED PERVIOUS LIFTING SURFACE THEORY:
MAIN MOTION OF THE SYSTEM PARALLEL TO ITS PLANE.

§2 *Winglet structures.*

The system in which we are interested consists of a number of airfoils, not necessarily straight, lying approximately in one plane. (e.g. boomerang, rotor of a helicopter). Such a system moves through a fluid (e.g. air, water), which is originally at rest with respect to an inertial frame I.F. and which, for our purposes, can be assumed to extend to infinity.

We introduce a right-handed cartesian coordinate system (x,y,z), which moves at a uniform velocity V in the negative x-direction with respect to I.F. through the fluid. (Hence the (x,y,z)-system itself is an inertial frame). The airfoils lie approximately in the (x,y)-plane, their projections on this plane remain within a region S which is fixed with respect to the (x,y,z)-system (see fig. 2.1). In addition to this main motion the airfoils may have velocities with respect to the (x,y,z)-system, which differ from place to place. The airfoils may also change

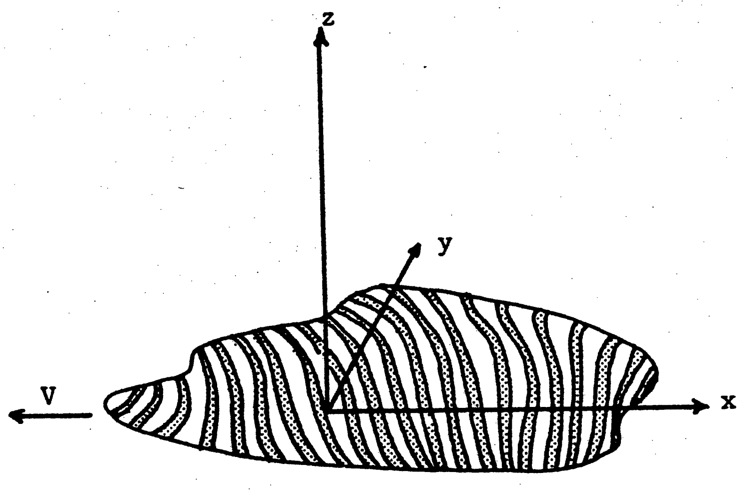


fig. 2.1. Structure of airfoils.

their shapes and their angles of incidence. However, they should not cross or overtake each other.

Next we introduce a local right-handed cartesian coordinate system (1,2,3), which is related to the local orientation of the airfoils (see fig. 2.2). The 1-direction is parallel to the (x,y)-plane, pointing from the local leading edge towards the local trailing edge of an airfoil. The 2-direction is parallel to the local spanwise direction of an airfoil. The spanwise direction of the airfoil deviates from the (x,y)-plane by a small angle γ. The 3-direction deviates from the z-direction by the same angle γ. The angle β is defined by identifying the projection on the (x,y)-plane of the 2-direction with the vector ($\cos\beta$, $\sin\beta$, 0) as notated in the (x,y,z)-system.

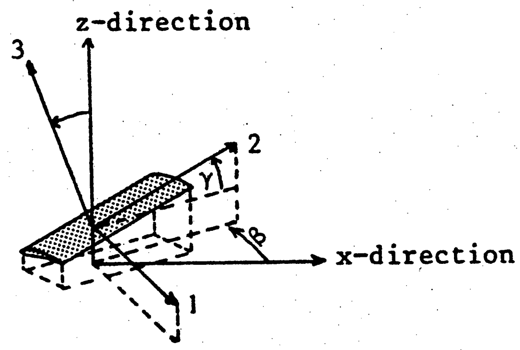


fig. 2.2. The local coordinate system (1,2,3).

The transformation matrix for the rotation from the (x,y,z)-system to the (1,2,3)-system is:

$$\begin{bmatrix} \sin\beta & -\cos\beta & 0 \\ \cos\beta\cos\gamma & \sin\beta\cos\gamma & \sin\gamma \\ -\cos\beta\sin\gamma & -\sin\beta\sin\gamma & \cos\gamma \end{bmatrix} \tag{2.1}$$

Next we introduce the "relative velocity" $\vec{V}_r(x,y)$, which is the local velocity of the fluid relative to the airfoils:

$$\vec{V}_r = \vec{v} + \vec{V} - \vec{W} \tag{2.2}$$

where

$\vec{v} = (v_x, v_y, v_z)$ is the local velocity of the fluid with respect to I.F.,

$-\vec{V} = -(V,0,0)$ is the velocity of the (x,y,z)-system with respect to I.F.,

$\vec{W} = (W_x, W_y, W_z)$ is the local velocity of the airfoils with respect to the (x,y,z)-system.

The components of these velocities are notated in the (x,y,z)-system. The components of $\vec{V}_r$ in the (1,2,3)-system are:

$$\left.\begin{aligned}
V_{r1} &= (V - W_x)\sin\beta + W_y\cos\beta + \\
&\quad + \{v_x\sin\beta - v_y\cos\beta\} \\
V_{r2} &= [(V - W_x)\cos\beta - W_y\sin\beta]\cos\gamma + \\
&\quad + \{(-W_z + v_z)\sin\gamma + (v_x\cos\beta + v_y\sin\beta)\cos\gamma\} \\
V_{r3} &= + \{-[(V - W_x)\cos\beta + W_y\sin\beta]\sin\gamma + (-W_z + v_z)\cos\gamma + \\
&\quad - (v_x\cos\beta + v_y\sin\beta)\sin\gamma\}
\end{aligned}\right\}\quad(2.3)$$

By definition of the 1-direction we have $V_{r1} \geq 0$ always. The function $\beta(x,y)$ may have jumps of magnitude π at points where $V_{r1} = 0$, and at these points an individual airfoil may suddenly have its leading edge turned into a trailing edge and reversely.

Further we introduce the local "effective angle of incidence" of the airfoils, $\alpha(x,y)$, which is the sum of the geometrical angle of incidence

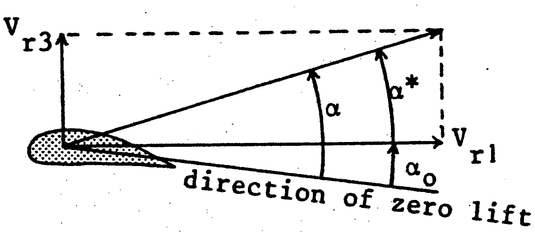


fig. 2.3. Angle of incidence.

with respect to the direction of zero lift $\alpha_o(x,y)$ and the angle:

$$\alpha^*(x,y) = \arctan \frac{V_{r3}}{V_{r1}} \qquad (2.4)$$

(see fig. 2.3).

We assume that the interaction between fluid and airfoils is determined by the following conditions:

A) The airfoils cannot locally exert or experience forces parallel to their local spanwise direction, i.e. the 2-direction.

B) The airfoils experience a local force in the (1,3)-plane, normal to the direction of $\vec{V}_r$, i.e. (in connection with A) parallel to the vector $(-V_{r3},0,V_{r1})$. The magnitude l of this force per unit of length along an airfoil, is given by

$$l(x,y) = \tfrac{1}{2}\mu(V_{r1}^2 + V_{r3}^2)\, b(x,y)\, C_L(x,y) \qquad (2.5)$$

where μ is the density of the fluid, b is the local chordlength of the airfoils and C_L their local lift coefficient.

C) C_L is taken to be proportional to $\sin\alpha$:

$$C_L(x,y) \;(:)\; \sin\alpha(x,y) = \sin[\alpha_o(x,y) + \alpha^*(x,y)] \qquad (2.6)$$

For a thin airfoil in two-dimensional flow the factor of proportionality theoretically is 2π. [Abbot, von Doenhoff, 1959].

These conditions mean that we apply a strip theory to the airfoils. The assumptions A, B, C appear to be valid for a real straight airfoil of large aspect ratio placed in a uniform flow, provided that the angle between the spanwise direction and the flow direction is not too small, and that effects due to viscosity may be neglected. [Abbot, von Doenhoff, 1959]. We expect that the assumptions will hold approximately in cases where the geometry of an airfoil does not vary much within spanwise distances comparable to its chordlength. Generally, the conditions will not be satisfied everywhere on S. Exceptions are for instance the regions close to the wingtips of the airfoils and the region where the airfoils are attached to an axle or a central disk.

The aerodynamic problem presented by the systems as they are described so far would be rather difficult to solve mathematically. A time-dependent case would be considerably simpler to deal with. This is our motive for approximating the structures of airfoils by the following model.

The region S is devided into subregions s. We introduce the local "filling factor" d as the fraction of s which is occupied by the airfoils' projections. We shall replace d, α_o, β, γ, $\vec{W}$ by continuous functions of (x,y) by replacing every airfoil by a large number of similar airfoils with smaller chordlengths, in such a way that the sum of their chordlengths remains unchanged. In the "limit" we name such "infinitesimal airfoils": *winglets*. Such a system of winglets exerts forces upon the fluid which will be distributed continuously over S: $\vec{f}(x,y)$ per unit of area. In general this field of forces $\vec{f}$ will be time-dependent. However, we shall confine our attention to those cases only where $\vec{f}$ is independent of time. Hence we only consider cases where d(x,y), $\alpha_o(x,y)$, $\beta(x,y)$, $\gamma(x,y)$, $\vec{W}(x,y)$ are *time-independent*.

From a somewhat different - and perhaps more rewarding - point of view we could say: the original airfoils have been "smeared out", so that at each point the original situation has been replaced by a kind of time-average. The winglet model thus would correspond to a sort of time exposure of the original airfoils. If the original system was periodic in time, its period should be small compared to the characteristic time D/V, where D is the diameter of S. Thus even a two-armed boomerang could be simulated by a winglet model, provided that it would spin fast enough. A theory based on the winglet model, of course, would at most yield information on *average* forces and torques acting on the original airfoil structure, and on *average* velocities of the fluid.

As far as the interaction with the fluid is concerned, only the stationary field of forces $\vec{f}(x,y)$ is of importance, and a steady flow results. For the magnitude of the forces per unit of area acting on the fluid we have, taking into account (2.6):

$$f(x,y) = -\mu c(x,y)\sin\alpha(x,y)\ (V_{r1}^2(x,y) + V_{r3}^2(x,y)) \qquad (2.7)$$

where c depends on the local density and profile shape of the winglets:

$$c(x,y) = d(x,y) \frac{C_L(x,y)}{2\sin\alpha(x,y)} \tag{2.8}$$

For thin airfoils we would obtain theoretically (see remark under C):

$$c(x,y) = \pi . d(x,y) \tag{2.9}$$

We remark that the condition

$$\vec{f} \perp \vec{V}_r \tag{2.10}$$

which is part of condition B, follows from a general argument, which is valid if viscosity is absent. Locally at the point (x,y,0) the winglets, per unit of area of S and per unit of time, do the work $\vec{f}(x,y).(\vec{W}(x,y) - \vec{V})$ on the fluid. On the other hand the fluid there wins, per unit of area and per unit of time, the energy $\vec{f}(x,y).\vec{v}(x,y)$. Therefore: $\vec{f}.(\vec{W}-\vec{V}) = \vec{f}.\vec{v}$ and $\vec{f} \perp (\vec{v} + \vec{V} - \vec{W})$.

The components of $\vec{f}$ in the (1,2,3)-system are:

$$\left.\begin{aligned} f_1 &= - f_3 \cdot \frac{V_{r3}}{V_{r1}} \\ f_2 &= 0 \\ f_3 &= - \mu c \sin\alpha \, V_{r1} \sqrt{V_{r1}^2 + V_{r3}^2} \end{aligned}\right\} \tag{2.11}$$

After substitution of (2.3) and (2.4), (2.11) shows how the forces $\vec{f}$ depend on the fluid's velocity $\vec{v}$. (2.11) gives a description of the interaction between winglets and fluid which is a property of the winglet structure. On the other hand in §3 a relation between $\vec{f}$ and $\vec{v}$ will be derived which is a property of the fluid. This relation will be a linear one, since we will linearize the equations of motion for the fluid. An exact, non-linear theory would be extremely difficult to work out.

Under which conditions is the application of the linearized theory to a winglet structure justified? The angle of incidence $\alpha(x,y)$ should be small, this means:

$$\left.\begin{array}{ll}\alpha_o(x,y) & \ll 1 \\ \gamma\ (x,y) & \ll 1 \\ W_z/\sqrt{(V-W_x)^2+W_y^2} & \ll 1\end{array}\right\} \quad (2.12)$$

The conditions can be formulated more exactly this way: Take a system with V, $\alpha_o(x,y)$, $\beta(x,y)$, $\gamma(x,y)$, $d(x,y)$, $\vec{W}(x,y)$ given. Now multiply α_o, γ and W_z by a small factor ε, and let ε tend to zero. Then α_o, γ, W_z are of $O(\varepsilon)$ and so will be v_x, v_y, v_z, f_z. However, f_x, f_y will be of $O(\varepsilon^2)$. The essence of the linearized theory is that quantities of $O(\varepsilon^{n+1})$ are neglected with respect to those of $O(\varepsilon^n)$.

In (2.3) the terms between braces are of $O(\varepsilon)$. We shall now linearize. The sign "$\approx$" between two expressions means that the relative difference between these expressions is of $O(\varepsilon)$. (2.3) becomes:

$$\left.\begin{array}{ll}V_{r1} \approx & (V-W_x)\sin\beta + W_y\cos\beta \\ V_{r2} \approx & (V-W_x)\cos\beta - W_y\sin\beta \\ V_{r3} \approx & -[(V-W_x)\cos\beta - W_y\sin\beta]\gamma - W_z + v_z\end{array}\right\} \quad (2.13)$$

We define the local "effective velocity" $V_e(x,y)$ by:

$$\sqrt{V_{r1}^2 + V_{r3}^2} \approx (V-W_x)\sin\beta + W_y\cos\beta \overset{\text{def}}{\equiv} V_e \quad (2.14)$$

And the angle α_e by:

$$\sin\alpha \approx \alpha_o + \frac{-[(V-W_x)\cos\beta - W_y\sin\beta]\gamma - W_z + v_z}{V_e}$$

$$\overset{\text{def}}{\equiv} \alpha_o + \alpha_e + \frac{v_z}{V_e} \quad (2.15)$$

The linearized version of (2.7) becomes:

$$f(x,y) \approx -\mu c(x,y)\left[\alpha_o(x,y) + \alpha_e(x,y) + \frac{v_z(x,y,0)}{V_e(x,y)}\right] V_e^2(x,y) \quad (2.16)$$

And (2.11) becomes:

$$\left.\begin{aligned} f_1 &\approx - f_3.(\alpha_e + \frac{v_z}{V_e}) \\ f_2 &= 0 \\ f_3 &\approx f \end{aligned}\right\} \quad (2.17)$$

The linearized components of f in the (x,y,z)-system are

$$\left.\begin{aligned} f_x &\approx + \mu c(\alpha_o + \alpha_e + \frac{v_z}{V_e}) V_e^2(\alpha_e + \frac{v_z}{V_e})\sin\beta \\ f_y &\approx - \mu c(\alpha_o + \alpha_e + \frac{v_z}{V_e}) V_e^2(\alpha_e + \frac{v_z}{V_e})\cos\beta \end{aligned}\right\} \quad (2.18)$$

$$f_z \approx - \mu c(\alpha_o + \alpha_e + \frac{v_z}{V_e}) V_e^2 \quad (2.19)$$

As far as the linearized theory is concerned the fluid is acted upon by the force field

$$\vec{f}(x,y) \approx (0,0,f_z(x,y)) \quad (2.20)$$

situated in the region S of the (x,y)-plane. The linearized fluid dynamics in §3 will lead to an integral relation of the form

$$v_z(x,y,0) \approx \frac{1}{4\pi\mu V} \iint_S K_o(x,y,\xi,\eta)\ f_z(\xi,\eta)d\xi d\eta \quad (2.21)$$

The equations (2.19) and (2.21) together constitute an integral equation for the function $f_z(x,y)$.

In some cases c, α_o, β, γ, $\vec{W}$ may be known beforehand, but in other cases some of these quantities may depend on the forces which the winglets experience from the fluid (as, for instance, in cases with elastic bending). In those cases one or more additional equations may be needed to determine the behaviour of the system. (Remember that we only consider systems giving rise to a steady flow). A simple example of such a system is discussed by Hess [1973, §3].

§3 *Linearized fluid dynamics.*

The behaviour of an ideal inviscid incompressible fluid is determined by the following two equations [Kotschin, Kibel, Rose, 1954], the equation of motion:

$$\mu \frac{d\vec{v}}{dt} = \vec{F} - \text{grad } p \tag{3.1}$$

and the equation of continuity:

$$\text{div } \vec{v} = 0 \tag{3.2}$$

The used symbols have the following meaning:

$\vec{v}$: velocity of the fluid,

p: pressure in the fluid,

μ: density of the fluid (constant),

$\vec{F}$: external forces per unit of volume acting on the fluid.

The operator $\frac{d}{dt}$ stands for $\frac{\partial}{\partial t} + \vec{v}.\nabla$

We shall use a right-handed cartesian coordinate system (x,y,z).

The left-hand side of (3.1) is not linear in $\vec{v}$, which makes an exact treatment of the fluid's behaviour very difficult. Therefore we shall work with a linear approximation. We assume $\vec{F}$ and $\vec{v}$ to be small of $O(\varepsilon)$ (for an explanation see §2), and we neglect quantities of $O(\varepsilon^{n+1})$ with respect to those of $O(\varepsilon^n)$. The linearized theory tends to be exact as ε tends to zero.

Taking the divergence of both sides of (3.1), we obtain

$$\mu \text{ div } \frac{d\vec{v}}{dt} = \text{div } \vec{F} - \text{div grad } p \tag{3.3}$$

Or, correct to $O(\varepsilon)$:

$$\text{div } \vec{F} = \text{div grad } p \tag{3.4}$$

This Poisson's equation yields for an infinite region:

$$p(x,y,z,t) = \iiint\limits_G - \frac{1}{4\pi r} \text{ div } F(\xi,\eta,\zeta,t)\, d\xi d\eta d\zeta \tag{3.5}$$

or the more convenient expression:

$$
\left.\begin{aligned}
p(x,y,z,t) &= \iiint_G \frac{\vec{F}(\xi,\eta,\zeta,t)\cdot\vec{r}}{4\pi r^3}\, d\xi d\eta d\zeta \\
\text{with} \qquad \vec{r} &= (x-\xi,\ \ y-\eta,\ \ z-\zeta)
\end{aligned}\right\} \quad (3.6)
$$

We assumed that $\vec{F}$ vanishes outside and on the boundary of a finite region G. From (3.6) it can be seen that the pressure field can be regarded as a superposition of pressure dipoles.

From now on we confine our attention to cases where a field of external forces $\vec{F}$ moves uniformly through a fluid which was originally at rest. The cartesian coordinate system (x,y,z) is chosen in such a way that its origin moves with the field of forces and the negative x-axis points in the direction of motion. With respect to this inertial frame we have a stationary field of forces in a uniform flow with a constant velocity V in the positive x-direction. This presents a steady flow problem.

If the velocity of the fluid with respect to this system is $\vec{V} + \vec{v}$, with $\vec{V} = (V,0,0)$ of $O(1)$ and $\vec{v} = (v_x, v_y, v_z)$ of $O(\varepsilon)$, the equation of motion (3.1) becomes

$$
(\vec{V}.\nabla)\vec{v} + (\vec{v}.\nabla)\vec{v} = \frac{1}{\mu}\vec{F} - \frac{1}{\mu}\,\text{grad } p \qquad (3.7)
$$

and the linearized equation of motion is:

$$
(\vec{V}.\nabla)\vec{v} = \frac{1}{\mu}\vec{F} - \frac{1}{\mu}\,\text{grad } p \qquad (3.8)
$$

We are particularly interested in fields of forces acting on a finite region S in the (x,y)-plane, with the forces directed parallel to the z-axis. In these cases the field of forces can be represented by:

$$
\left.\begin{aligned}
\vec{F}(x,y,z,t) &= \vec{f}(x,y)\delta(z) \\
\text{with} \qquad \vec{f} &= (0,0,f_z(x,y))
\end{aligned}\right\} \quad (3.9)
$$

where δ denotes Dirac's delta function. The pressure field (3.6) now becomes:

$$p(x,y,z) = \iint_S \frac{f_z(\xi,\eta)z}{4\pi r^3} d\xi d\eta$$

with

$$r = [(x-\xi)^2 + (y-\eta)^2 + z^2]^{\frac{1}{2}} \tag{3.10}$$

An expression for v_z on S can be obtained as follows. The z component of (3.8) is:

$$V \frac{\partial v_z}{\partial x} = \frac{1}{\mu} F_z - \frac{1}{\mu}\frac{\partial p}{\partial z} \tag{3.11}$$

Together with (3.9) and (3.10) this leads to

$$v_z(x,y,z) = \frac{1}{\mu V} \int_{-\infty}^{0} \left[f_z(x+\tau,y)\delta(z) - \right.$$

$$\left. - \frac{\partial}{\partial z} \iint_S \frac{z}{\{(x+\tau-\xi)^2+(y-\eta)^2+z^2\}^{3/2}} \frac{f_z(\xi,\eta)}{4\pi} d\xi d\eta \right] d\tau \tag{3.12}$$

Here we have used the conditions that f_z, $\frac{\partial p}{\partial z}$ and v_z vanish in the limit $x \to -\infty$. v_z is an even function of z and continuous at z=0: $v_z(x,y,0) = \lim_{z\to 0} v_z(x,y,z)$. For $z \neq 0$ the first term in the integrand of (3.12) vanishes, we then obtain by changing the order of the integrations and the differentiation:

$$v_z(x,y,z) = \frac{1}{4\pi\mu V} \iint_S \left[\int_{-\infty}^{0} - \frac{\partial}{\partial z} \frac{z}{\{(x+\tau-\xi)^2+(y-\eta)^2+z^2\}^{3/2}} d\tau \right] f_z(\xi,\eta) d\xi d\eta \tag{3.13}$$

This can be written as:

$$v_z(x,y,z) = \frac{1}{4\pi\mu V} \iint_S \overline{K}_o(x,y,z,\xi,\eta) f_z(\xi,\eta) d\xi d\eta \tag{3.14}$$

with

$$\overline{K}_o(x,y,z,\xi,\eta) = \frac{z^2-(y-\eta)^2}{[z^2+(y-\eta)^2]^2} \left\{ 1 + \frac{x-\xi}{[(x-\xi)^2+(y-\eta)^2+z^2]^{\frac{1}{2}}} \right\} +$$

$$+ \frac{z^2(x-\xi)}{[z^2+(y-\eta)^2][(x-\xi)^2+(y-\eta)^2+z^2]^{3/2}} \tag{3.15}$$

Substitution of z=0 into (3.15) yields:

$$K_o(x,y,\xi,\eta) \overset{\text{def}}{=\!=\!=} \overline{K}_o(x,y,0,\xi,\eta) = \frac{-1}{(y-\eta)^2}\left\{1 + \frac{x-\xi}{[(x-\xi)^2 + (y-\eta)^2]^{\frac{1}{2}}}\right\} \quad (3.16)$$

Obviously this kernel has a second order singularity for $\eta=y$ and $\xi \leqq x$. Because of this singular behaviour it is not permitted to simply substitute z=0 into (3.14) in order to obtain an expression for the induced velocity v_z on S. A careful analysis yields:

$$v_z(x,y,0) = \lim_{z\to 0} v_z(x,y,z) = \frac{1}{4\pi\mu V} \mathop{\not\!\not\!\int\!\!\int}_S K_o(x,y,\xi,\eta) f_z(\xi,\eta)\, d\xi d\eta \quad (3.17)$$

where $\not\!\not\!\int$ denotes the Hadamard principal value of the integral with respect to η, defined by

$$\not\!\not\!\int_a^b \frac{f(\eta)}{(y-\eta)^2} d\eta = \lim_{\beta\to 0}\left[\left\{\int_a^{y-\beta} + \int_{y+\beta}^b\right\} \frac{f(\eta)}{(y-\eta)^2} d\eta - \frac{2}{\beta} f(y)\right]$$

§4 *General character of the integral equation and its solution.*

In this section we shall investigate the general behaviour of the solution of the integral equation resulting from (3.17) and (2.19). It can be written in the form:

$$L(x,y) = P(x,y) - Q(x,y)\,\frac{1}{4\pi}\oint\!\!\int_S K_o(x,y,\xi,\eta)L(\xi,\eta)\,d\xi d\eta \qquad (4.1)$$

where $K_o(x,y,\xi,\eta)$ is given by (3.16) and where we have used the abbreviations:

$$\left.\begin{aligned} L &= \frac{-f_z}{\mu V^2} \\ P &= c(\alpha_o+\alpha_e)\,\frac{V_e^2}{V^2} \\ Q &= c\,\frac{V_e}{V} \end{aligned}\right\} \qquad (4.2)$$

The meaning of the symbols has been explained in §2. Note that $Q \geq 0$. (4.1) is an integral equation (with singular kernel) of the second kind, and it may be expected that the behaviour of its solution near the boundary of S will differ from the solution of the integral equation for an ordinary lifting surface, which is of the first kind.

First we shall investigate the behaviour of the solution in x-direction. Some essential information might already be obtained by studying the simple two-dimensional problem where the region S is a strip in y-direction: $-1 \leq x \leq 1$, and where P and Q do not depend on y. In this case (4.1), after integration with respect to η, reduces to:

$$L(x) = P(x) - Q(x)\,\frac{1}{4\pi}\oint_{-1}^{+1} K(x,\xi)L(\xi)\,d\xi \qquad (4.3)$$

with

$$K(x,\xi) = \frac{2}{x-\xi} \qquad (4.4)$$

The solution of this integral equation can be explicitly found [Muskhelishvili, 1953], but for the sake of simplicity we shall only consider the case where P and Q are constants. We then have:

$$L(x) = P - \tfrac{1}{2}Q.\frac{1}{\pi} \oint_{-1}^{+1} \frac{L(\xi)}{x-\xi}\, d\xi \qquad (4.5)$$

Its general solution is, provided that the singularities of L at $x = \pm 1$ are not stronger than those of $(1 \mp x)^{-1+\delta}$ with $\delta > 0$ [Muskhelishvili, 1953, Ch. 14]:

$$L(x) = \frac{P}{1+\frac{1}{4}Q^2} \left(\frac{1-x}{1+x}\right)^{\frac{1}{\pi}\arctan\frac{1}{2}Q} . \left\{1 + \frac{C}{1-x}\right\} \qquad (4.6)$$

where C is an arbitrary constant.
If Q tends to ∞ with P/Q remaining finite, we obtain the solution

$$L(x) = \frac{2P}{Q}\sqrt{\frac{1-x}{1+x}} . \left\{1 + \frac{C}{1-x}\right\} \qquad (4.7)$$

which corresponds to an ordinary two-dimensional flat plate placed in a homogeneous flow under an angle P/Q.

For each value of C (4.6) represents a different mathematical solution to the integral equation (4.5). A definite choice for C has to be made to obtain the unique solution corresponding to the physical system to which we want to apply the theory. In the case of ordinary lifting surfaces the choice

$$C = 0 \qquad (4.8)$$

is generally made. It corresponds to the Kutta condition: the trailing edge singularity vanishes. This condition is generally found to be in agreement with experiments. Further down in this section it is shown that porous lifting surfaces obey an integral equation similar to ours. It seems plausible that the Kutta condition would apply to these pervious lifting surfaces as well. We decide to make the choice (4.8) also for our winglet systems, but we recognize that its justification can be obtained from experiments only. We then have:

$$L(x) = \frac{P}{\sqrt{1+\frac{1}{4}Q^2}} \left(\frac{1-x}{1+x}\right)^{\frac{1}{\pi}\arctan\frac{1}{2}Q} \qquad (4.9)$$

Returning to the original three-dimensional equation (4.1), we can see that the behaviour of $L(x,y)$ near the boundary of S varies from place to place with $Q(x,y)$ hence with the local density of the winglets and their local effective velocity V_e.

There exists a remarkable analogy between winglet structures and porous lifting surfaces. With a normal non-porous lifting surface placed in a homogeneous flow the induced velocity must be such that the resulting flow is tangential to the lifting surface. With a porous lifting surface, however, there may be a leakage through it. If this leakage is taken proportional to the pressure difference between the two sides of the lifting surface, the induced velocity satisfies the equation:

$$\frac{v_z(x,y)}{V} = -\alpha(x,y) + \sigma(x,y)L(x,y) \tag{4.10}$$

Here α denotes the local angle of incidence and σ the local porosity coefficient. The other symbols have the same meaning as before. This leads to the integral equation

$$\sigma(x,y)L(x,y) = \alpha(x,y) - \frac{1}{4\pi} \oint\!\!\!\int_S K_o(x,y,\xi,\eta)L(\xi,\eta)d\xi d\eta \tag{4.11}$$

where $K_o(x,y,\xi,\eta)$ is the same as in (4.1). The two-dimensional case is treated by Barakat [1967].

Thus a winglet structure can be exactly simulated by a porous lifting surface with angle of incidence

$$\alpha(x,y) = \frac{P}{Q} = (\alpha_o+\alpha_e)\frac{V_e}{V} \tag{4.12}$$

and porosity coefficient

$$\sigma(x,y) = \frac{1}{Q} = \frac{V}{cV_e} \tag{4.13}$$

We see that the limit $Q \to \infty$ corresponds to a vanishing porosity coefficient, and the integral equation (4.1) tends to one of the first kind. On the other hand, in the limit $Q \to 0$ the integral equation tends to the algebraic equation:

$$L(x,y) = P(x,y) \tag{4.14}$$

In this case the induced velocity is of no importance. (In fact essentially (4.14) was used in a simple theory for boomerangs by Hess [1968]).

Because of the leading edge singularity a concentrated "suction" force of $O(\varepsilon^2)$ acting on the leading edge might exist. We assume this to be a local phenomenon, depending only on the local singularity in the lift distribution. Therefore the energy associated with "suction" forces taken locally from the fluid at the leading edge can easily be calculated by considering again the simple two-dimensional case. The total work done on the fluid in this case is zero. The energy taken from the fluid *at* the leading edge thus must equal the work done on the fluid *between* leading and trailing edge. This, per unit of time and per unit of length in y-direction, is given, correct to $O(\varepsilon^2)$, by

$$-\mu V^3 \int_{-1}^{+1} L(x) \frac{v_z(x)}{V} dx \tag{4.15}$$

$L(x)$ is given by (4.9) and $v_z(x)/V$ follows from

$$L = P + Q \frac{v_z}{V} \tag{4.16}$$

After substitution in (4.15) and integration we see that the integral vanishes if Q is finite. (Only in the non-porous case, $Q = \infty$, the integral in (4.15) generally does not vanish; in this case a suction force exists, which is a well known phenomenon.) Thus no suction forces act on the leading edges of porous lifting surfaces or winglet structures. However, in particular for high values of Q, one may expect peaks in the second order forces f_x and f_y near the leading edge. (The mistaken assertion in [Hess, 1973, §5] that suction forces are generally present if $Q > 0$ is based on an error in the calculation.)

Let us now consider the behaviour of the solution of the integral equation (4.1) in y-direction. Since we are particularly interested in cases where the region S is circular, we shall consider the case where S is a circle with radius 1.

Probably we can obtain some essential information as to the behaviour of the lift function near the "wingtips" $y = \pm 1$, by considering the simple case where $L(\xi,\eta) = L(\eta)$, independent of ξ. The integral in (4.1) then becomes:

$$\oint\!\!\!\!\!\!\!\!\!\!\!\!\;\;\;\int_{-1}^{+1} \frac{L(\eta)}{(y-\eta)^2} \int_{-\sqrt{1-\eta^2}}^{+\sqrt{1-\eta^2}} \left\{1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right\} d\xi d\eta \qquad (4.17)$$

We choose $x=0$. Then the integral with respect to ξ is simply equal to $2\sqrt{1-\eta^2}$ and (4.17) becomes:

$$\oint_{-1}^{+1} \frac{L(\eta).2\sqrt{1-\eta^2}}{(y-\eta)^2} d\eta = - \oint_{-1}^{+1} \frac{d\Gamma(\eta)}{d\eta} \frac{d\eta}{y-\eta} \qquad (4.18)$$

where we have put

$$2\sqrt{1-\eta^2}\, L(\eta) = \Gamma(\eta) \qquad (4.19)$$

and made use of the assumption that $\Gamma(1) = \Gamma(-1) = 0$. (4.1) now becomes (for $x=0$):

$$\frac{\Gamma(y)}{2\sqrt{1-y^2}} = P(0,y) + Q(0,y) \frac{1}{4\pi} \oint_{-1}^{+1} \frac{d\Gamma(\eta)}{d\eta} \frac{d\eta}{y-\eta} \qquad (4.20)$$

This integro-differential equation is of the type which is examined by Muskhelishvili [1953, Ch. 17]. By using the results given by him, it is easy to show that, in the special case in which $P(0,y)$ and $Q(0,y)$ are constants

$$\Gamma(y) = 2.\sqrt{1-y^2}.L(y) = 0(\sqrt{1-y^2}) \qquad (4.21)$$

for y close to ± 1. We assume that (4.21) will generally hold if $P(x,y)$ and $Q(x,y)$ are functions corresponding to actual winglet structures.

Finally it may be remarked that the method for solving the integral equation (4.1) outlined in the following sections obviously can also be applied to porous lifting surfaces satisfying equation (4.10).

§ 5 *The collocation method.*

From now on we shall consider systems where the region S is circular. The structure of winglets performs a translational motion and a rotational motion around an axis through the centre of the circle parallel to the z-axis. A system like this is described by the equations (3.17) and (2.19). In our problem the motion of the winglets is given and the forces on the winglets are to be found.

We start from equation (3.17), which for a circular region S with radius a takes the form

$$4\pi\mu V\ v_z(x,y,0) = \oint_{-a}^{+a} \int_{-\sqrt{a^2-\eta^2}}^{+\sqrt{a^2-\eta^2}} \frac{-f_z(\xi,\eta)}{(y-\eta)^2}\left\{1+\frac{x-\xi}{[(x-\xi)^2+(y-\eta)^2]^{\frac{1}{2}}}\right\}d\xi d\eta \tag{5.1}$$

$$4\pi v_z'(x',y',0)=$$

$$= \oint_{-1}^{+1} \int_{-\sqrt{1-\eta'^2}}^{+\sqrt{1-\eta'^2}} \frac{-f_z'(\xi',\eta')}{(y'-\eta')^2}\left\{1+\frac{x'-\xi'}{[(x'-\xi')^2+(y'-\eta')^2]^{\frac{1}{2}}}\right\}d\xi' d\eta' \tag{5.2}$$

where

$$\left.\begin{aligned} &x' = x/a\ ,\quad y' = y/a,\quad \xi' = \xi/a,\quad \eta' = \eta/a,\\ &v_z' = v_z/V,\quad f_z' = f_z/\mu V^2 \end{aligned}\right\} \tag{5.3}$$

are dimensionless quantities. For convenience we shall drop the primes from now on and write: x,y,ξ,η,v_z,f_z.

We introduce the following symbols:

$$\left.\begin{aligned} X' &= \frac{\xi}{\sqrt{1-\eta^2}}\ ,\quad X = \frac{x}{\sqrt{1-y^2}}\\ \tilde{X} &= \frac{x}{\sqrt{1-\eta^2}} = X\,\frac{\sqrt{1-y^2}}{\sqrt{1-\eta^2}}\\ Y &= \frac{y-\eta}{\sqrt{1-\eta^2}} \end{aligned}\right\} \tag{5.4}$$

X' and X will be used as new variables instead of ξ and x. In (X',η)-space the circular region S is transformed into a square.

We expand $f_z(\xi,\eta)$ in a series in ξ ("chordwise") direction with coefficients depending on the ("spanwise") coordinate η:

$$-f_z(\xi,\eta) = \sum_{p=0}^{M-1} a_p(\eta)H_p(X') \tag{5.5}$$

with:

$$\left.\begin{aligned} H_p(X') &= \sin p\phi, \quad p = 1,2,\ldots,M-1 \\ H_o(X') &= \tfrac{1}{2}(\pi-\phi), \quad p = 0 \\ X' &= -\cos\phi \end{aligned}\right\} \tag{5.6}$$

The function $\frac{1}{2}(\pi-\phi)$ replaces the customary $\operatorname{cotg}\frac{1}{2}\phi$ term of the Birnbaum series. The $\operatorname{cotg}\frac{1}{2}\phi$ guarantees the correct behaviour of the load function near the leading edge of an ordinary lifting surface, while each of the terms yield the correct behaviour near the trailing edge in that case. Unfortunately, we cannot make such a claim for the series expansion (5.6) in our case. On the other hand, since the behaviour of the load function generally varies from place to place near the boundary of S (as explanined in §4), it is impossible to obtain a series expansion of the type (5.5) which would yield the correct behaviour everwhere near the boundary of S. We found by actual numerical calculations that the customary series with the $\operatorname{cotg}\frac{1}{2}\phi$ term gave satisfactory results only in cases with large values of Q (as could be expected, see §4). Omitting the $\operatorname{cotg}\frac{1}{2}\phi$ term, however, gave rise to solutions with a tendency to build up a peak near the "leading edge". This peak, in particular for higher values of Q, could only be formed at the cost of a sometimes strongly oscillating solution. The choice of the function $\frac{1}{2}(\pi-\phi)$ is a compromise which seems to give reasonable results.

The functions $a_p(\eta)$ are expanded as

$$a_p(\eta) = \sum_{l=1}^{N} a_{pl}G_l(\eta) \tag{5.7}$$

with

$$\left.\begin{aligned} G_l(\eta) &= \frac{\sin l\vartheta}{\sin\vartheta} \qquad l = 1,\dots,N \\ \eta &= -\cos\vartheta \end{aligned}\right\} \tag{5.8}$$

It will be convenient to use the functions $h_l(\eta)$ defined by:

$$h_l(\eta) = G_l(\eta)\sqrt{1-\eta^2} = \sin l\vartheta \tag{5.9}$$

Substitution of (5.7) into (5.5) yields the series expansion for $f_z(\xi,\eta)$:

$$-f_z(\xi,\eta) = \sum_{l=1}^{N}\sum_{p=0}^{M-1} a_{pl}G_l(\eta)H_p(X') \tag{5.10}$$

which is in accordance with (4.21).

After substitution of (5.5), (5.9) and (5.4) into (5.2) we obtain

$$v_z(x,y,0) = \sum_{l=1}^{N}\sum_{p=0}^{M-1} a_{pl}v_{pl}(X,y) \tag{5.11}$$

with

$$v_{pl}(X,y) = \frac{1}{4\pi}\oint_{-1}^{+1}\frac{h_l(\eta)}{(y-\eta)^2}\int_{-1}^{+1}\left(1+\frac{\tilde{X}-X'}{[(\tilde{X}-X')^2+y^2]^{\frac{1}{2}}}\right)H_p(X')dX'd\eta \tag{5.12}$$

We now have

$$\sum_{l=1}^{N}\sum_{p=0}^{M-1} a_{pl}H_p(X)G_l(y) = P(X,y)+Q(X,y)\sum_{l=1}^{N}\sum_{p=0}^{M-1} a_{pl}v_{pl}(X,y) \tag{5.13}$$

as an approximation to the original integral equation (4.1). Our method for solving this equation will not be essentially different from Multhopp's collocation method [Multhopp, 1950]. We choose N×M pivotal points (X_μ,y_ν), $(\mu = 1\dots M,\ \nu = 1\dots N)$ and demand that (5.13) be satisfied at each of these points. There results a set of N×M linear algebraic equations, which can be solved for the N×M unknown coefficients

a_{pl}, ($p = 0 \ldots M-1$, $l = 1 \ldots N$).

The computation of the N^2M^2 elementary induced velocities $v_{pl}(X_\mu, y_\nu)$ (one for each pivotal point for each term in the series expansion) represents the bulk of the work required by this method.

§6 *The elementary induced velocities.*

Expression (5.12) has to be brought into such a form that it can be numerically evaluated at the pivotal points. The expansion of the integral with respect to X' in (5.12) near $\eta=y$ contains a term

$$-\left(\frac{dH_p}{dX'}\right)_X (y-\eta)^2 \ln|y-\eta| \tag{6.1}$$

Therefore we write (5.12) in the form

$$4\pi\, v_{p1}(X,y) = \oint_{-1}^{+1} \frac{h_1(\eta)}{(y-\eta)^2}\left[\int_{-1}^{+1}\left\{1+\frac{\tilde{X}-X'}{[(\tilde{X}-X')^2+Y^2]^{\frac{1}{2}}}\right\} H_p(X')\,dX' + \right.$$

$$\left. +\left(\frac{dH_p}{dX'}\right)_X \frac{(y-\eta)^2}{1-y^2}\ln|y-\eta|\right] d\eta - \left(\frac{dH_p}{dX'}\right)_X \frac{1}{1-y^2}\int_{-1}^{+1} h_1(\eta)\ln|y-\eta|\,d\eta \tag{6.2}$$

The expression between rectangular brackets is twice differentiable with respect to η at $\eta=y$.

Introducing the functions:

$$\left.\begin{aligned} f_p(X,y,\eta) &= \int_{-1}^{+1}\left\{1+\frac{\tilde{X}-X'}{[(\tilde{X}-X')^2+Y^2]^{\frac{1}{2}}}\right\} H_p(X')\,dX' + \\ &\quad +\left(\frac{dH_p}{dX'}\right)_X \frac{(y-\eta)^2}{1-y^2}\ln|y-\eta| \\ g_p(X,y) &= -\frac{1}{1-y^2}\left(\frac{dH_p}{dX'}\right)_X \end{aligned}\right\} \tag{6.3}$$

we can write (6.2) as

$$4\pi\, v_{p1}(X,y) = \oint_{-1}^{+1} \frac{h_1(\eta)}{(y-\eta)^2} f_p(X,y,\eta)\,d\eta + g_p(X,y)\int_{-1}^{+1} h_1(\eta)\ln|y-\eta|\,d\eta \tag{6.4}$$

Next we define the function R_p by:

$$f_p(X,y,\eta) = f_p(X,y,y,) + (\eta-y)\left(\frac{\partial f_p(X,y,\eta)}{\partial\eta}\right)_y + R_p(X,y,\eta) \qquad (6.5)$$

Near $\eta=y$ $R_p(X,y,\eta)$ contains a factor $(y-\eta)^2$ and

$$S_p(X,y,\eta) \overset{\text{def}}{=\!=\!=} \frac{R_p(X,y,\eta)}{(y-\eta)^2} \qquad (6.6)$$

is continuous at $\eta=y$. In [Hess, 1973, §8] an expression is derived for S_p in the limit $\eta \to y$:

$$\left.\begin{aligned}
S_p(X,y,y) &= \frac{1}{1-y^2}\left[\frac{1+2y^2}{1-y^2}\, XH_p(X) + \right. \\
&\left. + \left(\frac{dH_p}{dX'}\right)_X \left\{\frac{y^2}{1-y^2}X^2 + \tfrac{1}{2} + \ln[2(1-X)\sqrt{1-y^2}]\right\} + T_p(X)\right] \\
&\text{with} \\
T_p(X) &= \int_X^1 \left[\left(\frac{dH_p}{dX'}\right)_{X'} - \left(\frac{dH_p}{dX'}\right)_X\right]\frac{dX'}{X'-X} + \tfrac{1}{2}p\pi\,\frac{H_p(X)}{\sqrt{1-X^2}} \quad ,p = 1\ldots M-1 \\
&= \left(\frac{dH_o}{dX'}\right)_X \ln[2(1+X)] + \frac{\pi}{4(1+X)} \quad ,p = 0
\end{aligned}\right\} \quad (6.7)$$

Further we have

$$\left.\begin{aligned}
f_p(X,y,y) &= 2\int_{-1}^{X} H_p(X')\,dX' \\
\left(\frac{\partial f_p(X,y,\eta)}{\partial\eta}\right)_y &= \frac{2y}{1-y^2}\, XH_p(X)
\end{aligned}\right\} \quad (6.8)$$

The first expression follows directly from (6.3) and the second one is derived in [Hess, 1973, §8].

Now we can write (6.4) as:

$$4\pi\, v_{pl}(X,y) = f_p(X,y,y) \oint_{-1}^{+1} \frac{h_1(\eta)}{(y-\eta)^2}\, d\eta + \left(\frac{\partial f_p(X,y,\eta)}{\partial \eta}\right)_y \oint_{-1}^{+1} \frac{h_1(\eta)}{\eta-y}\, d\eta +$$

$$+ g_p(X,y) \int_{-1}^{+1} h_1(\eta) \ln|y-\eta|\, d\eta + \int_{-1}^{+1} h_1(\eta) S_p(X,y,\eta)\, d\eta \tag{6.9}$$

The first three integrals in (6.9) can be evaluated analytically, the last integral has to be evaluated numerically.

In order to avoid being forced to compute its integrand for η too close to y, we choose y itself as an integration point and write the integral as

$$\int_{-1}^{+1} S_p(X,y,\eta) h_1(\eta)\, d\eta = \left\{ \int_o^{\Theta} + \int_{\Theta}^{\pi} \right\} S_p(X,y,-\cos\vartheta) \sin l\vartheta\, \sin\vartheta\, d\vartheta \tag{6.10}$$

with

$$\eta = -\cos\vartheta, \quad y = -\cos\Theta \tag{6.11}$$

The integrand of (6.10) vanishes for $\vartheta=0$ and $\vartheta=\pi$.

As a final expression for the elementary induced velocity v_{pl} we now have:

$$v_{pl}(X,y) = \frac{1}{4\pi}\left[b_1(y) f_p(X,y,y) + c_1(y) \left(\frac{\partial f_p(X,y,\eta)}{\partial \eta}\right)_y + \right.$$

$$\left. + d_1(y) g_p(X,y) + \left\{ \int_o^{\Theta} + \int_{\Theta}^{\pi} \right\} S_p(X,y,-\cos\vartheta) \sin l\vartheta\, \sin\vartheta\, d\vartheta \right] \tag{6.12}$$

where $b_1(y)$, $c_1(y)$, $d_1(y)$ are given by:

$$\left.\begin{aligned}
b_1(y) &= -\pi l \frac{\sin l\Theta}{\sin\Theta} \\
c_1(y) &= \pi \cos l\Theta \\
d_1(y) &= \frac{\pi}{2}\left\{\frac{\cos(l+1)\Theta}{l+1} - \frac{\cos(l-1)\Theta}{l-1}\right\} \qquad l \geq 2 \\
&= \frac{\pi}{2}\left\{\tfrac{1}{2}\cos 2\Theta - \ln 2\right\} \qquad l = 1
\end{aligned}\right\} \tag{6.13}$$

We still have to substitute the expressions (5.6) for the functions $H_p(X')$ into the formulas. We put

$$X = -\cos\Phi \tag{6.14}$$

Then we have

$$\left.\begin{aligned}
\int_{-1}^{X} H_p(X')dX' &= \frac{1}{p^2-1}\,(\cos\Phi\ \sin p\Phi - p\ \sin\Phi\ \cos\Phi) && p=2\ldots M-1\\
&= \tfrac{1}{2}(\Phi - \tfrac{1}{2}\ \sin 2\Phi) && p=1\\
&= \tfrac{1}{2}(\pi-\sin\Phi-(\pi-\Phi)\cos\Phi) && p=0
\end{aligned}\right\} \tag{6.15}$$

The integral in the expression (6.3) for f_p has to be evaluated numerically, but it can be simplified by integration by parts:

$$\left.\begin{aligned}
&\int_{-1}^{+1} \left\{1 + \frac{\tilde{X}-X'}{[(\tilde{X}-X')^2+Y^2]^{\frac{1}{2}}}\right\} H_p(X')dX' =\\
&= p\int_{0}^{\pi} [(\tilde{X}+\cos\phi)^2+Y^2]^{\frac{1}{2}}\ \cos p\phi\ d\phi && p=2\ldots M-1\\
&= \frac{\pi}{2} + \int_{0}^{\pi} [(\tilde{X}+\cos\phi)^2+Y^2]^{\frac{1}{2}}\ \cos\phi\ d\phi && p=1\\
&= \frac{\pi}{2}\ \{1+[(\tilde{X}+1)^2+Y^2]^{\frac{1}{2}}\}-\tfrac{1}{2}\int_{0}^{\pi}[(X+\cos\phi)^2+Y^2]^{\frac{1}{2}}d\phi && p=0
\end{aligned}\right\} \tag{6.16}$$

§7 *The resulting forces and torques.*

In this section expressions are derived for the resulting forces and torques acting *on* the structure of winglets. We are particularly interested in the following six quantities (see fig. 7.1): the forces in x-, y- and z-directions: F_x, F_y, F_z, and the torques around the x-, y- and z-axes: T_x, T_y, T_z. The dimensionless equivalents of these, indicated

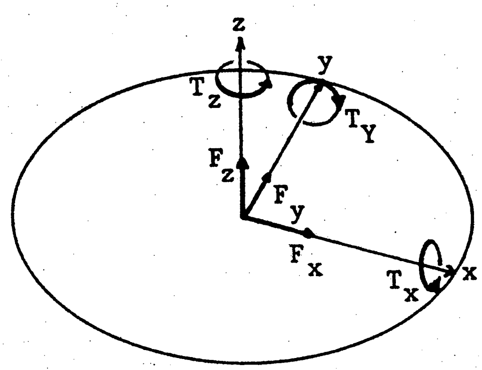


fig. 7.1. Forces and torques acting on system.

by a subscript o, will be evaluated. In order to obtain the actual forces and torques one should multiply the dimensionless quantities by $\mu V^2 a^2$ and $\mu V^2 a^3$ respectively. (a is the radius of the winglet structure). The lift:

$$F_{oz} = \iint_S - f_z(x,y)dxdy \tag{7.1}$$

The torque due to the lift distribution, with components:

$$T_{ox} = \iint_S - f_z(x,y)ydxdy \tag{7.2}$$

and

$$T_{oy} = \iint_S + f_z(x,y)xdxdy \tag{7.3}$$

The force in the (x,y)-plane, with components:

$$F_{ox} = \iint_S - f_x(x,y)dxdy \tag{7.4}$$

and

$$F_{oy} = \iint_S - f_y(x,y)dxdy \tag{7.5}$$

where f_x and f_y are defined by (2.18).
And the torque around the z-axis:

$$T_{oz} = \iint_S \{+ f_x(x,y)y - f_y(x,y)x\}dxdy \tag{7.6}$$

In the first three of these integrals we substitute the series expansion for $f_z(x,y)$ as given by (5.5) through (5.10):

$$F_{oz} = \sum_{l=1}^{N} \sum_{p=0}^{M-1} a_{pl} \int_{-1}^{+1} h_l(y) \int_{-1}^{+1} H_p(X)\,dXdy \tag{7.7}$$

$$T_{ox} = \sum_{l=1}^{N} \sum_{p=0}^{M-1} a_{pl} \int_{-1}^{+1} h_l(y)y \int_{-1}^{+1} H_p(X)\,dXdy \tag{7.8}$$

$$T_{oy} = \sum_{l=1}^{N} \sum_{p=0}^{M-1} -a_{pl} \int_{-1}^{+1} h_l(y)\sqrt{1-y^2} \int_{-1}^{+1} H_p(X)X\,dXdy \tag{7.9}$$

After evaluation of the integrals we obtain:

$$F_{oz} = \frac{1}{4}\pi^2(a_{o1}+a_{11}) \tag{7.10}$$

$$T_{ox} = -\frac{1}{8}\pi^2(a_{o2}+a_{12}) \tag{7.11}$$

$$T_{oy} = -\pi \sum_{\substack{l=1 \\ l\ \mathrm{odd}}}^{N} \frac{(\frac{1}{2}a_{o1}+a_{21})}{(l-2)l(l+2)} \tag{7.12}$$

When the integral equation for the load function $f_z(x,y)$ is solved by the method outlined in the preceding sections, the induced velocity v_z can be calculated either according to the algebraic equation (2.19) or according to the integral equation (3.17). The results will differ, since the solution for f_z is not exact. Only at the pivotal points there is agreement, and only there we can apply (2.18). In order to evaluate the integrals in (7.4), (7.5) and (7.6) we shall expand the functions $f_x(x,y)$ and $f_y(x,y)$ in series which are quite analogous to the expansion (5.10) for $f_z(x,y)$. The coefficients are determined by the requirement that the series have the calculated values at the N.M pivotal points. Thus we put:

$$-f_x(x,y) = \sum_{l=1}^{N} \sum_{p=0}^{M-1} b_{pl} G_l(y) H_p(X) \tag{7.13}$$

and

$$-f_y(x,y) = \sum_{l=1}^{N} \sum_{p=0}^{M-1} c_{pl} G_l(y) H_p(X) \tag{7.14}$$

These equations have to be satisfied at each of the N.M pivotal points, where f_x and f_y are known. There result two sets of N.M linear algebraic equations which can be solved for the unknown coefficients b_{pl} and c_{pl}.

If we accept these series expansions for $f_x(x,y)$ and $f_y(x,y)$ we obtain:

$$F_{ox} = \sum_{l=1}^{N} \sum_{p=0}^{M-1} b_{pl} \int_{-1}^{+1} h_l(y) \int_{-1}^{+1} H_p(X)\,dX\,dy \tag{7.15}$$

$$F_{oy} = \sum_{l=1}^{N} \sum_{p=0}^{M-1} c_{pl} \int_{-1}^{+1} h_l(y) \int_{-1}^{+1} H_p(X)\,dX\,dy \tag{7.16}$$

$$T_{oz} = \sum_{l=1}^{N} \sum_{p=0}^{M-1} \left\{ -b_{pl} \int_{-1}^{+1} h_l(y)\,y \int_{-1}^{+1} H_p(X)\,dX\,dy + \right.$$

$$\left. + c_{pl} \int_{-1}^{+1} h_l(y)\sqrt{1-y^2} \int_{-1}^{+1} H_p(X)\,X\,dX\,dy \right\} \tag{7.17}$$

The integrals can be evaluated, just like those for the lift distribution, and we obtain:

$$F_{ox} = \frac{1}{4}\pi^2 (b_{o1} + b_{11}) \tag{7.18}$$

$$F_{oy} = \frac{1}{4}\pi^2 (c_{o1} + c_{11}) \tag{7.19}$$

$$T_{oz} = \frac{1}{8}\pi^2 (b_{o2} + b_{12}) + \pi \sum_{\substack{l=1 \\ l\ \text{odd}}}^{N} \frac{(\frac{1}{2}c_{o1} + c_{21})}{(l-2)\,l\,(l+2)} \tag{7.20}$$

CHAPTER II.

LINEARIZED PERVIOUS LIFTING SURFACE THEORY: MAIN MOTION OF THE SYSTEM AT AN ANGLE ψ WITH ITS PLANE.

§8 *Linearized fluid dynamics.*

In this chapter we will consider the case where a field of external forces pointing in the z-direction is moving through an ideal fluid with a velocity having a component in z-direction. The field of forces is the same as the one considered in §3. The conditions under which this model can be applied to winglet structures are discussed in §9.

Again we choose a cartesian coordinate system (x,y,z) in which the field of forces is at rest, and we have a steady flow problem.

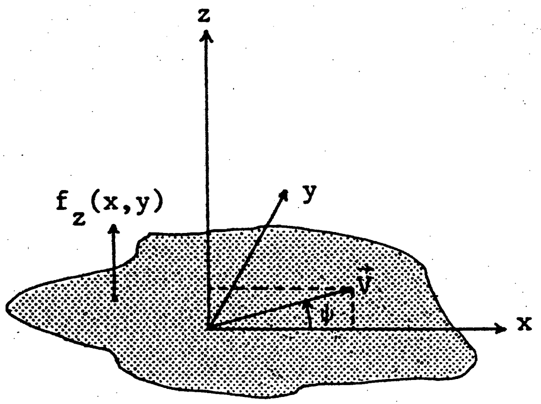


fig. 8.1. The system considered in this chapter.

Let the undisturbed velocity of the fluid with respect to the (x,y,z)-system be:

$$\left.\begin{array}{l} \vec{V} = V.(\cos\psi, 0, \sin\psi) \\ \text{with} \\ V > 0,\ 0 < |\psi| \leqq \pi/2 \end{array}\right\} \quad (8.1)$$

and the disturbance velocity field:

$$\vec{v} = (v_x, v_y, v_z)$$

The linearized equation of motion in this case is again given by (3.8). The equation for the z-direction is now:

$$V \cos\psi \frac{\partial v_z}{\partial x} + V \sin\psi \frac{\partial v_z}{\partial z} = \frac{1}{\mu} F_z - \frac{1}{\mu} \frac{\partial p}{\partial z} \tag{8.2}$$

The meaning of the symbols is the same as in §3.
Again we consider the case where the field of forces has the form (3.9). The pressure field p is then given by (3.10). Integration of (8.2) yields:

$$v_z(x,y,z) = \frac{1}{\mu V} \int_{-\infty}^{0} \Bigg[f_z(x+\tau\cos\psi, y)\delta(z+\tau\sin\psi) -$$

$$- \frac{\partial}{\partial z} \iint_S \frac{z+\tau\sin\psi}{\{(x+\tau\cos\psi-\xi)^2+(y-\eta)^2+(z+\tau\sin\psi)^2\}^{3/2}} \frac{f_z(\xi,\eta)}{4\pi} d\xi d\eta \Bigg] d\tau \tag{8.3}$$

Here we have used the conditions that f_z, $\frac{\partial p}{\partial z}$ and v_z vanish in the limit $(x,y,z) \to -\infty.(\cos\psi, 0, \sin\psi)$. The first term of the right-hand side in (8.3) yields:

$$\left.\begin{array}{ll} \dfrac{1}{\mu V|\sin\psi|} f_z(x - z\operatorname{cotg}\psi, y) & \text{if } z.\sin\psi > 0 \\ \text{and} & \\ \qquad\qquad 0 & \text{if } z.\sin\psi < 0 \end{array}\right\} \tag{8.4}$$

In the second term of the right-hand side in (8.3) we change the order of the integrations and the differentiation:

$$\frac{1}{4\pi\mu V} \iint_S \Bigg[\int_{-\infty}^{0} - \frac{\partial}{\partial z} \frac{z+\tau\sin\psi}{\{(x+\tau\cos\psi-\xi)^2+(y-\eta)^2+(z+\tau\sin\psi)^2\}^{3/2}} d\tau \Bigg] f_z(\xi,\eta) d\xi d\eta \tag{8.5}$$

The expression between brackets in (8.5) equals

$$\overline{K}_\psi(x,y,z,\xi,\eta) = \frac{z^2}{(\rho+\sigma)\rho^3} + \frac{z^2}{(\rho+\sigma)^2\rho^2} + \frac{(x-\xi)\cos\psi - z\sin\psi}{(\rho+\sigma)^2\rho} - \frac{\cos^2\psi}{(\rho+\sigma)^2} \tag{8.6}$$

with:

$$\rho = \sqrt{(x-\xi)^2+(y-\eta)^2+z^2}, \quad \sigma = -(x-\xi)\cos\psi - z\sin\psi \tag{8.6a}$$

We need an expression for $v_z(x,y,0)$. Substitution of $z=0$ in (8.6) leads to

$$K_\psi(x,y,\xi,\eta) \stackrel{\text{def}}{=\!=\!=} \overline{K}_\psi(x,y,0,\xi,\eta) =$$

$$= \frac{-\cos\psi}{\{\sqrt{(x-\xi)^2+(y-\eta)^2}-(x-\xi)\cos\psi\}^2}\left(\cos\psi - \frac{(x-\xi)}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right) \tag{8.7}$$

This kernelfunction, however, has a singularity for $\xi=x$, $\eta=y$. Therefore we shall consider $v_z(x,y,z)$ for $z\neq 0$ and approach the plane $z=0$ from the upstream side, where the kernelfunction has no singularities. Hence if $\sin\psi > 0$ we approach from the side with $z < 0$ and conversely.

We only need to evaluate the integral with respect to ξ and η in (8.5) over a region G of sufficiently small diameter containing the point $\xi=x, \eta=y$. For the rest of the region S we can simply substitute $z=0$. It turns out that the limit

$$\lim_{\substack{z\to 0\\ z\sin\psi<0}} \iint_G \overline{K}_\psi(x,y,z,\xi,\eta) f_z(\xi,\eta)\,d\xi d\eta \tag{8.8}$$

over a small region G containing the point (x,y) depends on the shape of G. [Hess, 1973, §12]. If G is a strip in ξ-direction with vanishing width, (8.8) yields:

$$\frac{2\pi}{|\sin\psi|}\, f_z(x,y) \tag{8.9}$$

and if G is a strip in η-direction with vanishing width:

$$2\pi|\sin\psi|\, f_z(x,y) \tag{8.10}$$

Hence we obtain as expressions for $v_z(x,y,0)$:

$$v_z(x,y,0) = \frac{f_z(x,y)}{2\mu V|\sin\psi|} + \frac{1}{4\pi\mu V} \oint\!\!\int_S K_\psi(x,y,\xi,\eta) f_z(\xi,\eta)\,d\xi d\eta \tag{8.11}$$

and

$$v_z(x,y,0) = \frac{f_z(x,y)\ |\sin\psi|}{2\mu V} + \frac{1}{4\pi\mu V}\oint\!\!\int_S K_\psi(x,y,\xi,\eta) f_z(\xi,\eta)\, d\eta\, d\xi \qquad (8.12)$$

It can easily be checked that for $\psi=0$ K_ψ reduces to the kernel of the integral equation in chapter I, K_o, as given by (3.16).

In the special case $\psi=\pm\frac{1}{2}\pi$ $K_\psi(x,y,\xi,\eta)$ vanishes and both (8.11) and (8.12) reduce to

$$v_z(x,y,0) = \frac{f_z(x,y)}{2\mu V} \qquad (8.13)$$

In this section we shall discuss under which conditions the theory of the preceding section can be applied to the structures of winglets described in §2. As before the infinite fluid is originally at rest with respect to an inertial frame I.F. The winglets occupy the two-dimensional region S, which is fixed with respect to the (x,y,z)-system. This coordinate system moves at the velocity $-\vec{V} = -(V\cos\psi, 0, V\sin\psi)$ with respect to I.F. We use the local coordinate system (1,2,3) which was introduced in §2, and the symbols α, β, γ, d, c. The angles β and γ are shown in fig. 2.2, α is the winglets' effective angle of incidence (see fig. 2.3), d is the local "filling factor", c follows from (2.8).

The local relative velocity of the fluid with respect to the winglets is:

$$\vec{V}_r = \vec{V} + \vec{v} - \vec{W} \tag{9.1}$$

where

$-\vec{V} = -(V\cos\psi, 0, V\sin\psi)$ is the velocity of the (x,y,z)-system with respect to I.F.

$\vec{v} = (v_x, v_y, v_z)$ is the local velocity of the fluid with respect to I.F.

$\vec{W} = (W_x, W_y, W_z)$ is the local velocity of the winglets with respect to the (x,y,z)-system.

The components of $\vec{V}_r$ in the (1,2,3)-system are:

$$\left.\begin{aligned}
V_{r1} &= (V\cos\psi - W_x)\sin\beta + W_y\cos\beta + \\
&\quad + \{v_x \sin\beta - v_y\cos\beta\} \\
V_{r2} &= [(V\cos\psi - W_x)\cos\beta - W_y\sin\beta]\cos\gamma + \\
&\quad + \{(V\sin\psi - W_z + v_z)\sin\gamma + (v_x\cos\beta + v_y\sin\beta)\cos\gamma\} \\
V_{r3} &= V\sin\psi\cos\gamma + \\
&\quad + \{-[(V\cos\psi - W_x)\cos\beta + W_y\sin\beta]\sin\gamma + (-W_z + v_z)\cos\gamma - \\
&\quad - (v_x\cos\beta + v_y\sin\beta)\sin\gamma\}
\end{aligned}\right\} \tag{9.2}$$

Remember that by definition of the 1-direction: $V_{r1} \geq 0$ always.

The effective angle of incidence $\alpha(x,y)$ is the sum of the geometrical angle of incidence $\alpha_o(x,y)$ and the angle $\alpha^*(x,y)$ as given by (2.4). According to the assumptions A, B, C of §2, the force per unit of area $\vec{f}$ in the direction $(-V_{r3},0,V_{r1})$, which is exerted by the winglets on the fluid, has the (positive or negative) magnitude $f(x,y)$ which is given by (2.7). The components of $\vec{f}$ in the (1,2,3)-system are again given by (2.11).

For a linear theory to be strictly correct it is required that the angle of incidence $\alpha(x,y)$ be small of $O(\varepsilon)$. This means that the following relation should hold:

$$\operatorname{tg} \alpha_o + \frac{V_{r3}}{V_{r1}} = O(\varepsilon) \tag{9.3}$$

For rigid winglet structures this condition generally cannot be satisfied on a considerable part of S, except for values of ψ close to zero. For those special cases where (9.3) is approximately satisfied a correct linear theory could be developed. After substitution of (9.2) into (2.11) and transformation to the (x,y,z)-system, we would obtain expressions for f_x, f_y, f_z in which the induced velocities v_x, v_y, v_z occur. We would then not only need an expression for v_z like (8.11) or (8.12) but also similar expressions for v_x and v_y. There would result three coupled integral equations, roughly requiring nine times as much work to be treated as the integral equation which we shall handle in this chapter.

If α_o and ψ would be of $O(\varepsilon)$, (9.3) would be satisfied, but in that case we could use the theory of chapter I. On the other hand, if ψ would not be small enough for the theory of chapter I to be used, α^* might still be small enough on the greater part of S for (9.3) to be reasonably approximated. It is to those cases that we intend to apply the theory of section 11. (A rapidly spinning boomerang with $\psi = 15^\circ$ might be an experimental example). Thus we suppose that

$$V_{r3}/V_{r1} \ll 1 \tag{9.4}$$

although ψ may not be small compared to one radian.

we then have:

$$\sqrt{V_{r1}^2 + V_{r3}^2} \approx (V\cos\psi - W_x)\sin\beta + W_y \cos\beta \overset{\text{def}}{=\!=\!=} V_e \tag{9.5}$$

and

$$\sin\alpha \approx \alpha_o + \frac{-[(V\cos\psi - W_x)\cos\beta + W_y\sin\beta]\gamma + V\sin\psi - W_z + v_z}{V_e} \overset{\text{def}}{=\!=\!=} \alpha_o + \alpha_e + \frac{v_z}{V_e} \tag{9.6}$$

which are the equivalents of (2.14) and (2.15). Hence we obtain:

$$\left.\begin{aligned} f_x &\approx +\mu c(\alpha_o + \alpha_e + \frac{v_z}{V_e})V_e^2(\alpha_e + \frac{v_z}{V_e})\sin\beta \\ f_y &\approx -\mu c(\alpha_o + \alpha_e + \frac{v_z}{V_e})V_e^2(\alpha_e + \frac{v_z}{V_e})\cos\beta \end{aligned}\right\} \tag{9.7}$$

$$f_z \approx -\mu c(\alpha_o + \alpha_e + \frac{v_z}{V_e})V_e^2 \tag{9.8}$$

which is similar to (2.18) and (2.19).

Substitution of expression (8.11) or (8.12) for the induced velocity $v_z(x,y,0)$ into (9.8) results in an integral equation for the load function $f_z(x,y)$. This integral equation can be numerically solved by the collocation method outlined in §5.

§10 *General character of the integral equation and its solution.*

In §4 we investigated the behaviour of the solution of the integral equation of chapter I. In this section we will do the same for the integral equation formed by (9.8) and (8.12).

Again we consider the simple two-dimensional case where the region S is a strip in y-direction: $-1 < x < 1$, and where the physical situation does not depend on y. In this case (after integration with respect to η) expression (8.12) for the induced velocity reduced to

$$v_z(x) = \frac{f_z(x)\,|\sin\psi|}{2\mu V} + \frac{\cos\psi}{2\pi\mu V}\oint_{-1}^{+1}\frac{f_z(\xi)}{x-\xi}\,d\xi \tag{10.1}$$

If we use the abbreviations:

$$\left.\begin{aligned} L &= \frac{-f_z}{\mu V^2} \\ P &= \frac{c(\alpha_o+\alpha_e)\frac{V_e^2}{V^2}}{1+\frac{1}{2}c\,|\sin\psi|\frac{V_e}{V}} \\ Q &= \frac{c\,\frac{V_e}{V}\cos\psi}{1+\frac{1}{2}c\,|\sin\psi|\frac{V_e}{V}} \end{aligned}\right\} \tag{10.2}$$

we can write the integral equation for the two-dimensional case in the form:

$$L(x) = P(x) - Q(x)\,\frac{1}{2\pi}\oint_{-1}^{+1}\frac{L(\xi)}{x-\xi}\,d\xi \tag{10.3}$$

This is exactly the same form as (4.3), and the observations made in §4 could be repeated here.

The limit case $Q \to \infty$ cannot occur since

$$\frac{1}{Q} = \frac{V}{cV_e\cos\psi} + \tfrac{1}{2}|\operatorname{tg}\psi| > \tfrac{1}{2}|\operatorname{tg}\psi| \tag{10.4}$$

On the other hand, the limit case $Q \to 0$ can be reached. Then (10.3) can be approximated by the algebraic equation

$$L(x) = P(x) \tag{10.5}$$

From (10.4) it is obvious that this approximation holds in any case for values of ψ close to $\pm \frac{\pi}{2}$, which is in agreement with (8.13).

We conclude that the solution of the integral equation (9.8) will behave the same way as the one of chapter I, but that the leading edge singularity generally will have a decreasing intensity for increasing values of the angle ψ. As far as the behaviour in y-direction in concerned, we assume (4.21) to be valid again.

In the following sections we shall use the same series expansion for the load function $f_z(x,y)$ as in chapter I.

§11 *The elementary induced velocities.*

The integral equation formed by (9.8) and (8.11) or (8.12) will be handled by the same methods that were used for the case $\psi=0$ in §5 and §6. In this section we shall deal with (8.11) rather than with (8.12), since the latter expression for the induced velocity leads to somewhat more complicated formulas [Hess, 1973, §17], and the numerical evaluation of the elementary induced velocities in this case requires more computing time than in the case where (8.11) is used [Hess, 1973, §25]. Both methods, however, yield identical numerical results.

For a circular region (8.11) takes the form

$$v_z(x,y,0) = \frac{f_z(x,y)}{2|\sin\psi|} + \frac{1}{4\pi} \oint_{-1}^{+1} \int_{-\sqrt{1-\eta^2}}^{+\sqrt{1-\eta^2}} K_\psi(x,y,\xi,\eta) f_z(\xi,\eta)\, d\xi d\eta \qquad (11.1)$$

with $K_\psi(x,y,\xi,\eta)$ from (8.7). Here, like §5, we have replaced the original quantities x, y, ξ, η, v_z, f by the dimensionless ones (5.3) and dropped the primes. As in chapter I, we shall use X', X, $\tilde{X}$ and Y, defined by (5.4).

Again the load function $f_z(\xi,\eta)$ is expanded in a series according to (5.5) through (5.10). The induced velocity v_z can be written as a sum of elementary induced velocities v_{pl} according to (5.11). Instead of (5.12) we now have:

$$v_{pl}(X,y) = -\frac{H_p(X)G_l(y)}{2|\sin\psi|} + \frac{1}{4\pi} \oint_{-1}^{+1} \int_{-\sqrt{1-\eta^2}}^{+\sqrt{1-\eta^2}} -K_\psi(x,y,\xi,\eta) H_p(X')G_l(\eta)\, d\xi d\eta$$

(11.2)

We integrate by parts with respect to ξ, and obtain for the integral in (11.2):

$$I_{pl}(x,y,0) = \oint_{-1}^{+1} \left\{ -IK(x,y,\xi,\eta)H_p(X')G_l(\eta) \Big|_{\xi=-\sqrt{1-\eta^2}}^{\xi=+\sqrt{1-\eta^2}} \right\} d\eta \; +$$

$$+ \oint_{-1}^{+1} \int_{-\sqrt{1-\eta^2}}^{+\sqrt{1-\eta^2}} IK(x,y,\xi,\eta) \frac{\partial}{\partial\xi} \{H_p(X')G_l(\eta)\} d\xi d\eta \qquad (11.3)$$

with

$$\left.\begin{aligned} IK\,(x,y,\xi,\eta) &= \frac{\cos\psi}{[(x-\xi)^2+(y-\eta)^2]^{\frac{1}{2}} - (x-\xi)\cos\psi} \\ \text{or} \qquad IK^*(X,y,X',\eta) &= \frac{\cos\psi}{[(\tilde{X}-X')^2+Y^2]^{\frac{1}{2}} - (\tilde{X}-X')\cos\psi} \cdot \frac{1}{\sqrt{1-\eta^2}} \end{aligned}\right\} \quad (11.4)$$

IK and K_ψ are related by

$$K_\psi(x,y,\xi,\eta) = \frac{\partial}{\partial\xi} IK(x,y,\xi,\eta) \qquad (11.5)$$

The first term in (11.3) vanishes for $p \geqq 1$, for p=0 it equals:

$$\frac{\pi}{2} \oint_{-1}^{+1} \frac{h_1(\eta)}{\sqrt{1-\eta^2}} IK^*(X,y,-1,\eta)d\eta \qquad (11.6)$$

The second term in (11.3) equals:

$$\oint_{-1}^{+1} \frac{h_1(\eta)}{\sqrt{1-\eta^2}} \int_{-1}^{+1} IK^*(X,y,X',\eta) \frac{dH_p}{dX'} dX'\, d\eta \qquad (11.7)$$

The expansion of the integral with respect to X' in (11.7) near $\eta=y$ contains a term

$$-\frac{2}{\sqrt{1-y^2}} \frac{\cos\psi}{\sin^2\psi} \left(\frac{dH_p}{dX'}\right)_X \ln|y-\eta| \qquad (11.8)$$

Therefore we write (11.7) in the form

$$\int_{-1}^{+1} h_1(\eta) f_p(X,y,\eta)d\eta + g_p(X,y) \int_{-1}^{+1} h_1(\eta)\ln|y-\eta|d\eta \qquad (11.9)$$

with

$$f_p(X,y,\eta) = \frac{1}{\sqrt{1-\eta^2}} \int_{-1}^{+1} IK^*(X,y,X',\eta) \frac{dH_p}{dX'} dX' + \frac{2}{1-y^2} \frac{\cos\psi}{\sin^2\psi} \left(\frac{dH_p}{dX'}\right)_X \ln|y-\eta|$$

(11.10)

$$g_p(X,y) = -\frac{2}{1-y^2}\frac{\cos\psi}{\sin^2\psi}\left(\frac{dH_p}{dX'}\right)_X \tag{11.10a}$$

The term (11.6) can be included in the expression for f_p if p=0,

$$\frac{\pi}{2}\frac{1}{\sqrt{1-\eta^2}} IK^*(X,y,-1,\eta) \tag{11.11}$$

In [Hess, 1973, §16] a derivation is given for f_p in the limit $\eta \to y$:

$$\left.\begin{aligned}
f_p(X,y,y) &= \frac{\cos\psi}{\sin^2\psi}\frac{1}{1-y^2}\left[\left(\frac{dH_p}{dX'}\right)_X\left\{2\ln[2(1-X)\sqrt{1-y^2}] + \cos\psi\ln\left(\frac{1-\cos\psi}{1+\cos\psi}\right)\right\}+\right.\\
&\left.+(1+\cos\psi)\pi p\frac{H_p(X)}{\sqrt{1-X^2}} - 2\int_X^1\left[\left(\frac{dH_p}{dX'}\right)_{X'} - \left(\frac{dH_p}{dX'}\right)_X\right]\frac{dX'}{X-X'}\right] \quad \text{if } p = 1\ldots M-1\\
f_o(X,y,y) &= \frac{\cos\psi}{\sin^2\psi}\frac{1}{1-y^2}\left[\left(\frac{dH_o}{dX'}\right)_X\left\{2\ln[2(1-X)^2\sqrt{1-y^2}] + \cos\psi\ln\left(\frac{1-\cos\psi}{1+\cos\psi}\right)\right\}+\right.\\
&\left.+\frac{\pi}{2}\frac{1+\cos\psi}{1+X}\right] \quad \text{if } p = 0.
\end{aligned}\right\} \tag{11.12}$$

Here (11.11) has been taken into account. The integrals in (11.10) and (11.12) have to be evaluated numerically. Now we can write v_{p1} as

$$v_{p1}(X,y) = -\frac{H_p(X)G_1(y)}{2|\sin\psi|} + \frac{1}{4\pi}\left\{\int_{-1}^{+1} h_1(\eta)f_p(X,y,\eta)d\eta + g_p(X,y)d_1(y)\right\} \tag{11.13}$$

where $d_1(y)$ is given by (6.13). The integral in (11.13) has to be evaluated numerically. We write it in the form

$$\left.\begin{aligned}
&\left\{\int_o^{\Theta} + \int_{\Theta}^{\pi}\right\} f_p(X,y,-\cos\vartheta)\sin l\vartheta\,\sin\vartheta\,d\vartheta\\
\text{with}\\
&y = -\cos\Theta
\end{aligned}\right\} \tag{11.14}$$

The integrand in (11.14) vanishes for $\vartheta=0$ and $\vartheta=\pi$.

The integral equation can now be solved by the collocation method and the resulting forces and torques acting on the winglet structure can be calculated according to §7.

CHAPTER III

NUMERICAL INTEGRATIONS AND DISTRIBUTION OF PIVOTAL POINTS.

§12 *The method of numerical integration.*

This section deals with the method used to evaluate numerically certain integrals occurring in the chapters I and II. For all of the integrations we use one and the same method, which works as follows. The integral:

$$\int_a^b f(x)\,dx \tag{12.1}$$

has to be evaluated with an absolute tolerance δ. The integration interval $[a,b]$ is divided into a number of subintervals. One such interval $[x_o,x_4]$, with $x_4-x_o = 4h$, is divided into four equal pieces by the points x_1, x_2, x_3. Simpson's rule, applied to the intervals $[x_o,x_2]$ and $[x_2,x_4]$, yields:

$$\int_{x_o}^{x_4} f(x)\,dx = \frac{1}{3}\,h(f_o+4f_1+2f_2+4f_3+f_4) - \frac{1}{45}\,h^5 f^{iv}(\xi_1) \tag{12.2}$$

Bode's rule, applied to the interval $[x_o,x_4]$, yields:

$$\int_{x_o}^{x_4} f(x)\,dx = \frac{2}{45}\,h(7f_o+32f_1+12f_2+32f_3+7f_4) - \frac{8}{945}\,h^7 f^{vi}(\xi_2) \tag{12.3}$$

where f_i stands for $f(x_i)$, and $x_o < \xi_1, \xi_2 < x_4$ [Hildebrand, 1956]. Subtracting (12.3) from (12.2) we obtain:

$$0 = \frac{1}{45}\,h(f_o-4f_1+6f_2-4f_3+f_4) - \frac{1}{45}\,h^5 f^{iv}(\xi_1) + \frac{8}{945}\,h^7 f^{vi}(\xi_2) \tag{12.4}$$

Using the abbreviation:

$$D = f_o-4f_1+6f_2-4f_3+f_4 \tag{12.5}$$

and neglecting the last term in (12.4), we have

$$\frac{1}{45} h^5 f^{iv}(\xi_1) \approx \frac{1}{45} h\, D = \frac{1}{180}(x_4 - x_o) D \tag{12.6}$$

We demand that

$$|D| < \frac{180}{|b-a|}\, \delta \tag{12.7}$$

for each of the intervals $[x_o, x_4]$. For then the absolute error in the integral (12.1), as computed according to Simpson's rule (12.2) with neglect of the last term, would be less than δ, provided we may neglect the last term in (12.4).

The integration procedure goes like this: If (12.7) is satisfied for the interval $[x_o, x_4]$, the integral over this interval is computed by Bode's rule (12.3) with neglect of the last term. However, if (12.7) is not satisfied, the interval $[x_o, x_4]$ is divided into two halves, and the process is repeated for each of these. By this method the fineness of the division of the integration interval is allowed to vary widely in different regions, depending on how strong the integrand there fluctuates.

As to the integrals which have to be integrated numerically, we shall first consider those of chapter I. In (6.12) these integrals occur:

$$\left\{ \int_o^{\Theta} + \int_{\Theta}^{\pi} \right\} S_p(X, y, -\cos\vartheta) \sin l\vartheta \sin\vartheta \, d\vartheta, \quad p = 0 \ldots M-1, \; l = 1 \ldots N \tag{12.8}$$

where (X,y) is successively each of the pivotal points. According to (6.5) and (6.6) the functions S_p contain the functions f_p, which in turn contain the integrals

$$\int_{-1}^{+1} \left\{ 1 + \frac{\tilde{X} - X'}{[(\tilde{X} - X')^2 + Y^2]^{\frac{1}{2}}} \right\} H_p(X')\, dX', \quad p = 0 \ldots M-1 \tag{12.9}$$

according to (6.3). These integrals were written in a different form in (6.16). It is these integrals (12.9) which consume by far the greater part of the time needed to compute the integrands in (12.8). However, the integrals (12.9) do not depend on l. Therefore it is advantageous to compute simultaneously the integrals (12.8) which differ only in l. Hence for each of the NM pivotal points (X,y) and for each value of p, *one* subdivision of the integration interval $0 \leq \vartheta \leq \pi$ is used for the computation of the set of integrals (12.8) with $l = 1 \ldots N$. For all of the N integrands the condition (12.7) has to be satisfied simultaneously. The integrals (12.9) have to be computed at each of the integration points in the interval $0 \leq \vartheta \leq \pi$ with a tolerance that should be so small that the tolerance requirement for the integrals (12.8) can be satisfied.

We now consider the integrals in chapter II. In (11.14) we have the integrals:

$$\left\{ \int_{0}^{\Theta} + \int_{\Theta}^{\pi} \right\} f_p(X,y,-\cos\vartheta)\sin l\vartheta \sin\vartheta \, d\vartheta, \quad p = 0 \ldots M-1, \; l = 1 \ldots N \qquad (12.10)$$

According to (11.10) the f_p contain the integrals

$$\int_{-1}^{+1} IK^{*}(X,y,X',\eta) \frac{dH_p}{dX'} \, dX', \quad p = 0 \ldots M-1 \qquad (12.11)$$

Here we can repeat the preceding paragraph, provided we replace (12.8) by (12.10) and (12.9) by (12.11).

The tolerances for each of the numerical integrations are determined as follows. We start by setting a certain absolute tolerance, TOL, for all of the elementary induced velocities $v_{pl}(X_\mu, y_\nu)$. From this follow the required tolerances for the "spanwise" integrals (12.8) and (12,10). As to the "chordwise" integrals (12.9) and (12.11), their tolerances will have to depend on the way they are contained in the "spanwise" integrands of (12.8) and (12.10). On the one hand, if their tolerances are taken too small, they will be computed with unnecessary accuracy, and too much computing time is used. On the other hand, if their tolerances are taken too large, the "spanwise" integrands are computed with insufficient accuracy. Then it might even be impossible to satisfy the tolerance requirement for the spanwise integrations at all. If the absolute tolerance for a "spanwise" integral is δ and the length of the integration interval is L, its integrand should be computed with an absolute tolerance less than δ/L. We determine the tolerances of the "chordwise" integrations in such a way that the resulting error in the "spanwise" integrand is less that $MT \times \delta/L$, where MT is a "margin of tolerance" with $0 < MT < 1$. Actual calculations suggested that MT = .2 is a reasonable choice.

The accuracy of the numerical solution to the integral equation is determined mainly by these two factors: First, the number NM of terms in the expansion for f_z and the distribution of the NM pivotal points. Secondly, the accuracy of the numerical integrations. With our method the second factor can be separated from the first one. In this our method of integration is rather different from the one used by Multhopp [1950]. There the spanwise integrations are performed by expanding the integrands in as many interpolation functions as there are pivotal points in spanwise direction. Thus the number of integration points for all spanwise integrals is fixed and equal to N. Zandbergen, Labrujere and Wouters [1967] adopt an intermediate method. They use $a(N+1) -1$ integration points, where a is a natural number, fixed for all spanwise integrations. (See also [Labrujere and Zandbergen, 1973]).

§13 *The distribution of the pivotal points.*

It is clear that the theory outlined in the chapters I and II allows an arbitrary distribution of pivotal points to be used, provided those points occupy a rectangular lattice in (X,y)-space. They are denoted by (X_μ, y_ν), $\mu = 1 \ldots M$, $\nu = 1 \ldots N$. In this section we shall make a definite choice for the distribution of pivotal points over the circular region S.

In ordinary lifting surface theory often Multhopp's distribution of pivotal points is chosen. Multhopp's chordwise distribution is based on the following assumptions [Multhopp, 1950]:

I. Two-dimensional flow.

II. Given is the shape of the airfoil, hence the induced velocity; to be determined is the lift distribution.

III. The exact solution can be expanded in a (Birnbaum) series of M+1 terms.

And the following requirement:

IV. If M terms are used in the chordwise (Birnbaum) expansion of the lift function, the first M moments of the lift distribution have to be equal to those of the exact solution.

It can be shown that under these conditions the pivotal points

$$X_\mu = -\cos\Phi_\mu, \quad \mu = 1 \ldots M \tag{13.1}$$

have to be chosen in such a way that

$$\tfrac{1}{2} + \sum_{q=1}^{M} \sin q\Phi_\mu = 0 \tag{13.2}$$

is satisfied for each of them. This leads to Multhopp's chordwise distribution of pivotal points:

$$\Phi_\mu = \frac{\mu\pi}{M+\frac{1}{2}}, \quad \mu = 1 \ldots M \tag{13.3}$$

Some critical remarks can be made:

1º Assumption III is essential. It is justified if in the Birnbaum expansion of the exact solution the sum of the (M+2)-th and higher terms can be neglected with respect to the (M+1)-th term.

2º If the requirement stated in IV would be replaced by the following (seemingly not less reasonable) requirement:

IV* The first M coefficients in the Birnbaum series have to be equal to those of the exact solution,

we would easily find that the pivotal points should satisfy:

$$\cos M\Phi_\mu = 0, \quad \mu = 1\ldots M \tag{13.4}$$

This would lead to the symmetrical distribution:

$$\Phi_\mu = \frac{(\mu-\frac{1}{2})\pi}{M}, \quad \mu = 1\ldots M \tag{13.5}$$

which is quite different from Multhopp's distribution (13.3).

3º The reasoning cannot be applied to airfoils with small aspect ratios like, for instance, circular wings.

It is possible to set up a similar argument for our integral equation. In our case not the induced velocity would be given but the functions which in the sections 4, 5 and 10 have been denoted by P and Q. Assumption III would be unreasonable here, since we know that our chordwise expansion with the term $\frac{1}{2}(\pi-\phi)$ does not correspond to the behaviour of the exact solution. This behaviour near the leading edge is known, it depends on the function Q. For the two-dimensional case a reasonable assumption, taking the place of III, might be made, concerning the expansion of the exact solution. It is clear that the "optimal" distribution then would depend on the function Q. However, critical remarks like those made for Multhopp's case could be made here as well. It seems to us that there are no strong theoretical arguments in favor of any particular distribution of pivotal points. Except perhaps that it would be sensible to have a distribution which is more dense near the boundary of the lifting surface, or in any region where sharp variations in the lift distribution are expected.

As to the chordwise distribution of pivotal points, actual calculations [Hess, 1973, §24] possibly gave slight indications that, the smaller the function Q, the more the points should be shifted towards the leading edge. We found that the symmetrical distribution:

$$X_\mu = -\cos\Phi_\mu, \quad \Phi_\mu = \frac{\mu\pi}{M+1}, \quad \mu = 1\ldots M \tag{13.6}$$

works satisfactorily. As to the spanwise distribution, we followed Multhopp [1950] and took.

$$y_\nu = -\cos\Theta_\nu, \quad \Theta_\nu = \frac{\nu\pi}{N+1}, \quad \nu = 1\ldots N \tag{13.7}$$

Zandbergen, Labrujere and Wouters [1967] have tried some other spanwise distributions (for ordinary lifting surfaces), but no definite conclusions as to an "optimal" distribution could be made. A distribution based on (13.6) and (13.7) with N=6, M=6 is shown in fig. 13.1.

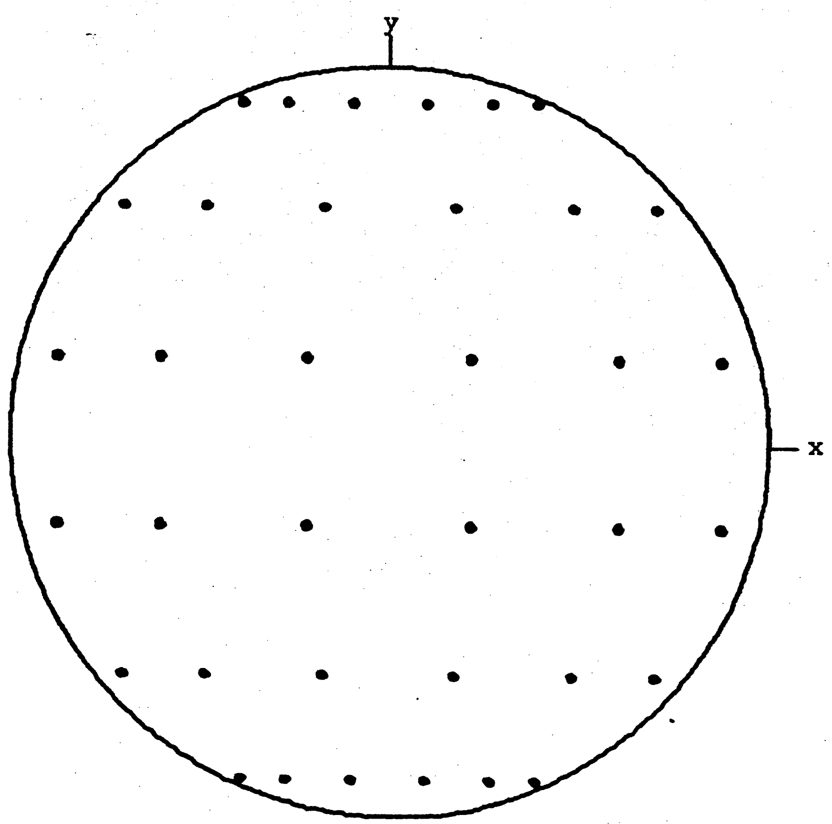


fig. 13.1. A possible distribution of pivotal points.

The symmetry of the spanwise distribution (13.7) is important with regard to the time consuming computation of the elementary induced velocities. This symmetry allows us to reduce by a factor of 2 the number of integrals which have to be computed numerically. A further reduction could be obtained by making use of the symmetry in the chordwise distribution (13.6).

CHAPTER IV

SOME NUMERICAL AND EXPERIMENTAL RESULTS.

§14 *Comparison with Van Spiegel's theory for circular wings.*

It would be useful to check the correctness of the mathematical theories developed in chapters I and II against other, independent, theories. This is indeed feasible for the case $\psi = 0$. (ψ is the angle between the plane of the winglet structure and the direction of the undisturbed flow.) Van Spiegel [1959] has developed a linearized lifting surface theory for ordinary circular wings, and has given some numerical results. It is easy to adapt our computing program for the case $\psi = 0$ (called program B) to ordinary lifting surfaces where the induced velocity v_z is given. However, because of the term $\frac{1}{2}(\pi-\varphi)$ instead of the customary term $\operatorname{cotg} \frac{1}{2}\varphi$ in the chordwise expansion of the lift function, our theory is not particularly suited to be used for wings with little or no porosity. Nonetheless a comparison with van Spiegel's results may be of interest.

We shall work with the dimensionless quantities defined by (5.3). Van Spiegel [1959] gives numerical results for six cases, in which the shape of the circular wing and hence the induced velocity is prescribed:

$$\left.\begin{array}{l} v_z = -1 \\ v_z = x \\ v_z = x^2 \\ v_z = y^2 \\ v_z = y \\ v_z = xy \end{array}\right\} \qquad (14.1)$$

For each of these cases numerical values are given for the components F_{oz}, T_{ox}, T_{oy} and F_{ox} (defined in §7). Because of symmetry, in the first four cases $T_{ox} = 0$, in the last two cases $F_{oz} = 0$ and $T_{oy} = 0$, whereas $F_{oy} = 0$ and $T_{oz} = 0$ in all six cases.

In table 14.1 van Spiegel's results [1959, p. 84/5] are compared with those of our program B. The pivotal points were chosen according to (13.6) and (13.7) with N=6, M=6 and N=8, M=8 respectively. The elementary induced velocities were computed with a tolerance TOL = .002. The agreement appears to be good: the relative differences are of the order of 1%. This warrants a certain confidence as to the correctness of the theory of chapter I within its own framework. We remark that an earlier version of program B with a cotg $\frac{1}{2}\varphi$ term instead of $\frac{1}{2}(\pi-\varphi)$ yielded results which agreed slightly better with van Spiegel's.

As to the drag component F_{ox} (not listed in table 14.1), program B yields values which are roughly a factor of 2 higher than van Spiegel's. This must be due to the fact that the suction forces acting on the leading edge are not taken into account by our method.

case	component	van Spiegel	Program B	
			6×6	8×8
$v_z = -1$	F_{oz}/π	+.8951	+.8836	+.8891
	$-T_{oy}/\pi$	-.4663	-.4661	-.4665
$v_z = x$	F_{oz}/π	-.4663	-.4737	-.4702
	$-T_{oy}/\pi$	-.2194	-.2196	-.2201
$v_z = x^2$	F_{oz}/π	-.3755	-.3709	-.3730
	$-T_{oy}/\pi$	-.0118	-.0116	-.0112
$v_z = y^2$	F_{oz}/π	-.2213	-.2186	-.2198
	$-T_{oy}/\pi$	+.0962	+.0958	+.0959
$v_z = y$	T_{ox}/π	-.1225	-.1211	-.1220
$v_z = xy$	T_{ox}/π	-.0576	-.0590	-.0584

table 14.1. Comparison between van Spiegel [1959] and our program B.

§15. *Accuracy of numerical results.*

In this section we consider the influence on the final numerical results of A: the tolerance chosen for the elementary induced velocities, and B: the distribution of pivotal points. Finally we compare the results of the theory of chapter I ($\psi = 0$) with those of the theory of chapter II ($\psi \neq 0$), for small angles ψ.

The examples which were calculated for this section are based on the theoretical winglet structures described in §18. These correspond to the experimental boomerangs used in the experiment described in §17, in particular to the boomerangs with 8 and 2 arms respectively, spinning at a reduced rotational velocity $\Omega \equiv \omega a/V = 2$. The computing programs used for the numerical computations are called B and BC respectively; B is based on the theory for $\psi = 0$, and BC on the theory for $\psi \neq 0$. These programs were written in Algol and run on a Telefunken TR4 computer.

A: *the tolerance for the elementary induced velocities.*

Here the pivotal points were chosen according to (13.6) and (13.7) with $N = 6$, $M = 6$. The "margin of tolerance" (see §12) was set as $MT = .2$ and the tolerance TOL for the elementary induced velocities was varied from .1 to .001. Computations were made both for $\psi = 0°$ and for $\psi = 30°$. It turns out that reducing TOL by a factor of 10 increases the computing time t for the elementary induced velocities by a factor of 4 roughly. Hence the following relation holds approximately:

$$t \sim \mathrm{TOL}^{-0.6} \tag{15.1}$$

From the tables given by Hess [1973, §23] it can be inferred that the relative errors in the computed six force and torque components are of the order of 1% for TOL = .02, and of the order of 0.1% for TOL = .002.

B: *the distribution of pivotal points.*

Here a tolerance TOL = .002 was used, so that the errors due to inaccuracies of the integrations were of the order of 0.1% only. The computations were made for $\psi = 0^\circ$ (program B).

First we consider some chordwise distributions of pivotal points which differ from (13.6). Instead we now take, just for experimental purposes:

$$X_\mu = -\cos\Phi_\mu, \quad \Phi_\mu = \frac{(\mu+e)\pi}{M+1}, \qquad \mu = 1\ldots M \tag{15.2}$$

with a variable parameter e. For $e=0$ (15.2) reduces to (13.6) again. The spanwise distribution is taken according to (13.7). We choose $N=6$, $M=6$ and $e=-0.4$, -0.2, 0, $+0.2$, $+0.4$ respectively. From the tables in [Hess, 1973, §24] it appears that for $-0.4 \leq e \leq +0.2$ the variations in F_{oz} and T_{ox} are of the order of ½%, while the variations in T_{oy} are larger: some 5% for the case with 8 arms and some 10% for the case with 2 arms. (The absolute variations are about the same in both cases). It is difficult to draw definite conditions concerning an "optimal" distribution, although the results for the case with 2 arms at $e=+.4$ deviate so strongly from the corresponding results with the other e values, that it seems advisable not to shift the points that far from the leading edge, in particular if the function Q belonging to the problem (see §4) is small.

A not too unreasonable criterion as to the correctness of a numerical solution might be based on the values of the coefficients a_{pl} of the expansion for the load function $f_z(x,y)$. The faster the absolute values of the a_{pl} decrease with increasing p and l, the smoother a solution will be. A solution, strongly oscillating with a period corresponding to that of a function $H_p(X)$ or $G_l(y)$ with $p=M-1$ or $l=N$, very probably would not resemble the exact solution. Although one should be careful, one might consider smoothness as a measure of quality for a solution. For a smooth solution the absolute values of the coefficients a_{pl} should decrease suffiently fast with increasing p and l. We take as a – somewhat arbitrary – measure of quality the quantity:

$$\overline{a} \stackrel{\text{def}}{=\!=\!=} \frac{1}{MN} \sum_{p=0}^{M-1} \sum_{l=1}^{N} (p+1)\, l\, |a_{pl}| \tag{15.3}$$

A smaller $\overline{a}$ would mean a smoother and hence "better" solution.

For both 8 arms and 2 arms $\bar{a}$ is minimal for $e \approx -0.1$ [Hess, 1973, §24]. For cases with higher values of Q, $\bar{a}$ is minimal for $e > 0$. This may be an indication that for an "optimal" distribution the pivotal points should be shifted the more towards the leading edge the smaller the function Q.

From now on we shall use distributions with $e = 0$ exclusively, i.e. distributions according to (13.6) and (13.7).
It is remarkable that the coefficients a_{pl} generally appear to have significantly higher absolute values in cases with odd M than in cases with even M. This is demonstrated by table 24.5 in [Hess, 1973, §24] which contains values for $\bar{a}$, as defined by (15.3). It seems therefore advisable to take the chordwise number of pivotal points, M, even.
The numerical differences between the cases with $N = 6$, $M = 6$ and $N = 8$, $M = 8$ and $N = 8$, $M = 6$ are of the order of 1% for the components F_{oz} and T_{ox} and somewhat greater for the component T_{oy}: roughly 2% for the case with 8 arms and 10% for the case with 2 arms. (The absolute differences are roughly the same in both cases). The components F_{ox}, F_{oy} and T_{oz} show larger relative differences but their absolute values are rather small.

For cases with $N = M$ the computing time t for the elementary induced velocities appears to be proportional to the third power of the number of pivotal points. A closer inspection [Hess, 1973, §24] learns that the following relation holds approximately

$$t \sim N^2 M^{3.7} \tag{15.4}$$

The number of integration points for a spanwise integration appears to be greater for elementary induced velocities associated with higher values of p and pivotal points closer to the edge of S. The computing program B with TOL = .002 and $N = 8$, $M = 8$ for instance leads to numbers of integration points varying from 7 (lowest possible number) to 131. Generally the pivotal points with $\nu = 1$ and $\mu = 1$ or M require the greatest number of integration points. This is true for program CB as well.

C: *comparison between programs B and CB for small angles* ψ.

If the elementary induced velocities computed for $\psi = 10^\circ$ and for $\psi = 5^\circ$ by program CB are extrapolated to $\psi = 0^\circ$, the extrapolated values are in good agreement with the elementary induced velocities for $\psi = 0^\circ$ as computed by program B. This shows that for $\psi \to 0$ the results of program CB would approach those of program B with $\psi = 0^\circ$.

For program CB we found that the computing time t for the elementary induced velocities satisfies the approximate relation:

$$t \sim |\sin\psi|^{-1} \tag{15.5}$$

Although the theory of chapter I was developed for the case $\psi = 0$, it can also be applied to cases with small values of ψ by taking, instead of $W_z = 0$,

$$W_z = -V \sin\psi \tag{15.6}$$

in (2.15). (cf. (9.6) with $W_z = 0$.) Thus, instead of the fluid having a small velocity in z-direction, the winglet structure as a whole has the opposite velocity. The differences between the results of the programs B and CB are then exclusively due to the difference in the direction of the undisturbed flow with respect to the plane of S. With a vorticity representation in mind, we could say that in the theory of chapter I (program B) the free vorticity which is released from the upstream part of S is carried with the undisturbed flow along S, and, remaining in the (x,y)-plane, along the downstream part of the winglets. In the theory of chapter II (program CB) however, the free vorticity is carried with the undisturbed flow at an angle ψ relative to the plane of S, so that hardly any vorticity passes close to the trailing edge of S. On this basis we would expect the computed lift function in the downstream part of S to be smaller according to B than according to CB. This is indeed confirmed by the numerical results [Hess, 1973, §25], computed with TOL = .002. At $\psi = 5^\circ$, for instance, and for 8 arms, F_{oz} is 5% lower, T_{ox} 16% lower and T_{oy} 9% higher according to B than according to CB. For 2 arms the corresponding differences are respectively 1%, 4% and 12%. At $\psi = 2\frac{1}{2}^\circ$ the differences are smaller, at $\psi = 10^\circ$ greater.

It might be inferred that it would not be justified to use program B for cases with $|\psi|$ greater than a few degrees. However, we cannot be sure that program CB would give more reliable results, because the computed induced velocities may well be such that the fluid is deflected over considerable angles from its original direction of flow. In such cases a linearized theory cannot be justified anyway, although the results might still be reasonable. Let us consider an example. For $\psi = 0^\circ$, 5° or 10° the computed induced velocity v_z is such that on a considerable part of S the fluid is deflected by an angle of about -10° for the case with 8 arms and about -5° for the case with 2 arms. It might therefore be possible that the case with 8 arms at $\psi = 10^\circ$ (or 2 arms at $\psi = 5^\circ$) could be handled better by program B than by program CB. This would be a simple way of roughly taking deviations from linearity into consideration. We shall, however, adhere to the linearized theory, and let the free vorticity drift in the direction of the undisturbed flow.

§16 *Pictures of some lift distributions.*

This section shows twelve computer graphs (fig. 16.1 and 16.2) which represent pictures of theoretical lift distributions computed for the winglet structures of §18. The cases chosen are those for which:

$$\left.\begin{aligned} n &= 8,\ 2 \\ \psi &= 0^\circ,\ 15^\circ \\ \Omega &= 1,\ 2,\ 5 \end{aligned}\right\} \quad (16.1)$$

(n = number of arms, $\Omega = \omega a/V$). The computations were made by program B ($\psi = 0^\circ$) and program CB ($\psi = 15^\circ$) with $N = 6$, $M = 6$, $TOL = .02$.

The graphs were produced in the following way. The circular region S was devided into 50 slices of equal width by cuts parallel to the x-direction. Each of the 49 "chords" again was devided into 50 equal pieces by 51 points including the endpoints. The lift function was computed at these points, and for each "chord" a graph was made by drawing a broken line. There resulted 49 graphs which together constitute a picture of the lift distribution on S.
In the pictures, the undisturbed flow is from the left to the right, and the boomerang's rotation is counterclockwise.

The pictures for 2 arms (fig. 16.2) show that an appreciable peak at the leading edge of S appears only for $\psi = 0^\circ$, $\Omega = 5$. This suggests that for two-armed boomerangs the $\frac{1}{2}(\pi-\varphi)$ term in the chordwise expansion of the load function could be omitted without much trouble. Even smoother solutions might be obtained this way, as long as Ω would not become too great; see §24 for a comparison.

The small-scale undulations in the graphs probably are partly artefacts. They might be altered by a different choice for the distribution of pivotal points, or by a different choice for N and M.

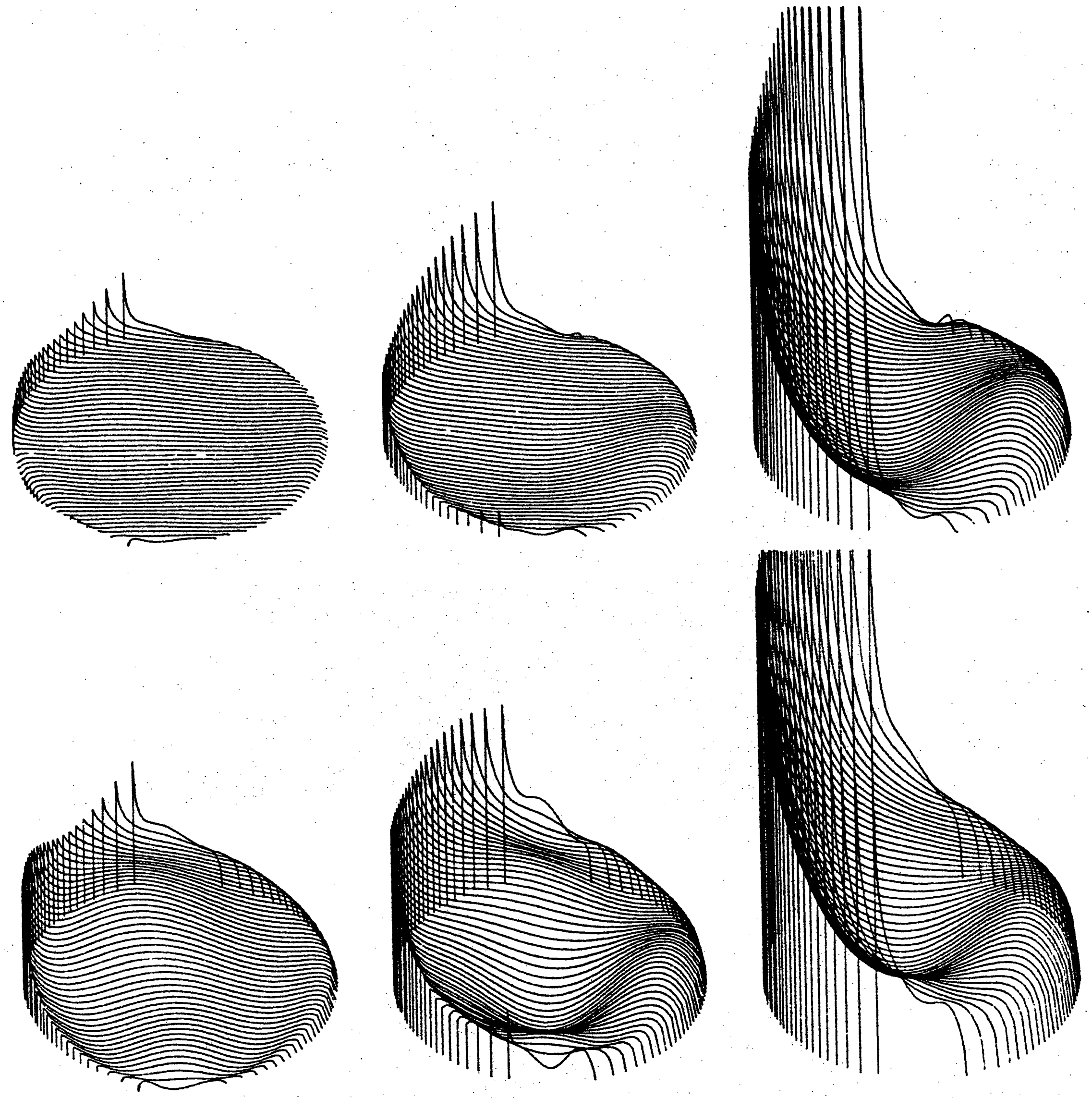


fig. 16.1 Theoretical lift distributions for 8-armed boomerang, computed with $N = 6$, $M = 6$.
Top row: $\psi = 0^\circ$, bottom row: $\psi = 15^\circ$. From left to right: $\Omega = 1,2,5$.
Undisturbed flow from left to right, boomerang's rotation counter-clockwise.
Vertical scale at right corresponds to load function $f_z(x,y) = 1$.

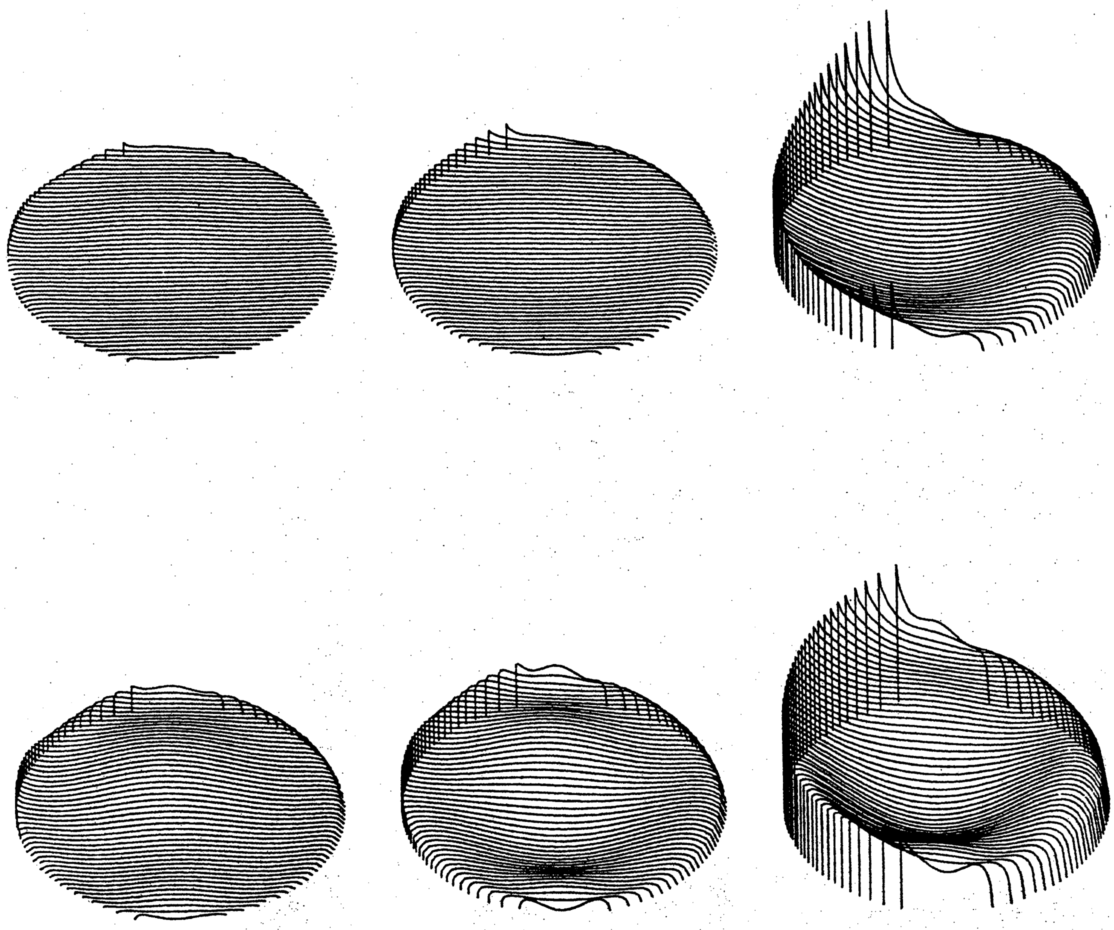


fig. 16.2 Theoretical lift distributions for 2-armed boomerang, computed with $N = 6$, $M = 6$.

Top row: $\psi = 0^\circ$, bottom row: $\psi = 15^\circ$. From left to right: $\Omega = 1,2,5$.

Undisturbed flow from left to right, boomerang's rotation counter-clockwise.

Vertical scale at right corresponds to load function $f_z(x,y) = 1$.

§17 *An experiment.*

In order to test the validity of the winglet model, an experiment was carried out in which time-averages were measured of the forces and torques on experimental boomerangs. This section describes the experimental set-up, §18 deals with the boomerang arms used, and in §19 the experimental results are given and compared with theory. The boomerangs used in this experiment (see fig. 17.1) consist of 8, 4 or 2 identical arms fixed together by two steel flanges. The boomerang is attached to a shaft through its geometrical centre (= centre of mass), by which it can be driven at certain angular velocities, while being towed under water at certain linear velocities.

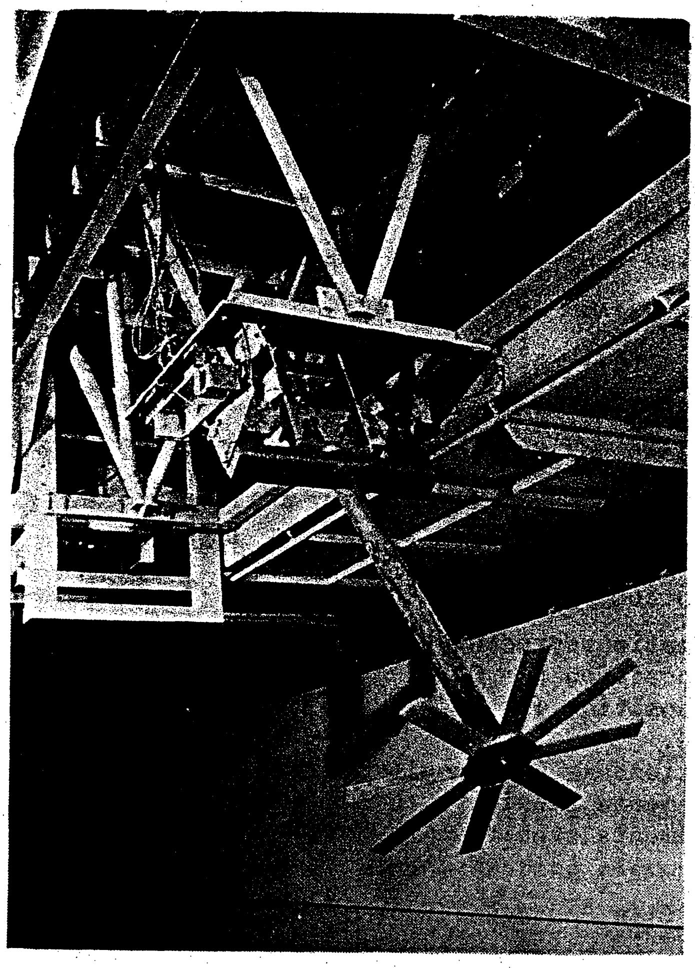


fig. 17.1. Measuring apparatus with 8-armed boomerang in the position $\psi = 30^\circ$ (without water).

A schematic representation of the measuring apparatus is shown in fig. 17.2. The boomerang (Ø 50 cm.) with its shaft together with the motor drive forms one unit, B. The forces exerted by the water on this body are determined by measuring the forces between unit B and the towing wagon which moves at a uniform speed. In this way possible inertial forces, due to vibrations of unit B, mix with the hydrodynamic forces. However, since we confine our interest to the time-averages of forces, and the output of the measuring elements is integrated with respect to time, the inertial forces are expected to cancel.

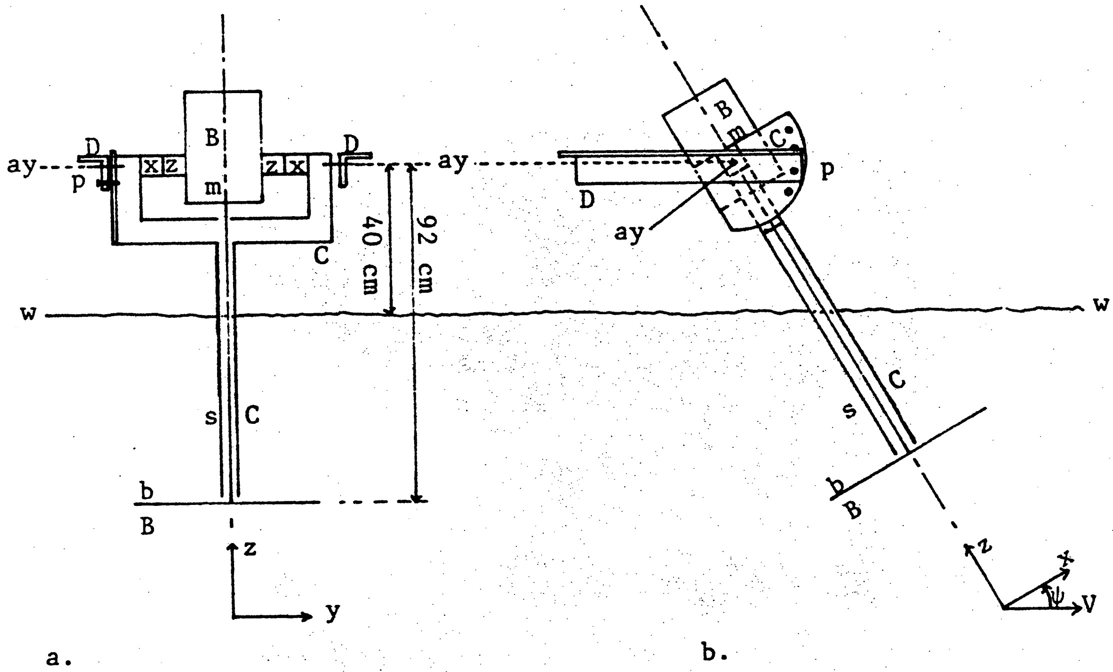


fig. 17.2 Schematic representation of measuring apparatus.
a) front view, position $\psi = 0^\circ$.
b) side view , position $\psi = 30^\circ$.
w: water surface.
V: towing direction.
x: measuring elements sensitive to forces in x-direction.
z: measuring elements sensitive to forces in z-direction.
b: boomerang.
m: motor.
s: streamlined pipe.
B: unit containing boomerang, shaft, motor drive.
C: part containing streamlined pipe.
D: frame fixed to towing wagon.
ay: axis about which B and C can be rotated to vary ψ.
p: pin to fix B and C in selected ψ position.

Directly attached to unit B are two measuring elements, which measure forces in axial or z-direction. To these are attached two elements which measure forces in x-direction. By means of these four elements unit B is connected to a frame C, to which a streamlined pipe is rigidly attached which shields the shaft from hydrodynamic forces. This pipe can be seen in fig. 17.1. Part C, together with B and the measuring elements, can be rotated about a horizontal axis in y-direction through the four measuring elements and through two bearings which connect C to the towing wagon. By changing the orientation of C about this axis, different values for ψ can be chosen. The distance between the measuring elements and the boomerang's centre is 92 cm. In the position $\psi = 0$ the boomerang is situated 52 cm. underneath the surface of the water. In the position $\psi = 30^{\circ}$ the boomerang's centre is 40 cm. below the water surface, and the wingtips between 27 cm. and 52 cm.

As our original intention was to measure all six force and torque components, the experimental set-up contained two additional measuring elements. These, however, were not situated between unit B and the streamlined pipe, but "outside" part C, so that they were also sensitive to hydrodynamic forces exerted on the pipe. The measurements were done with 8, 4 and 2 arms respectively. Measurements without any arms were also done to serve as corrections. However, the flow with boomerang arms may considerably differ from the flow with the central disk only. Because of this the forces on the streamlined pipe may well be different in the two cases. Hence, unfortunately, the output of the two additional measuring elements cannot be used, and only the output of the first four elements is useful. By means of these the three components F_{oz}, F_{ox} and T_{oz} can be determined.

The cubic measuring elements were developed and manufactured by the Shipbuilding Laboratory of the Delft University of Technology, where also the experiment was conducted. Each measuring element contains four steel leafsprings on which strain gauges (eight in all) are glued, the whole being contained in a cube with sides of 5 cm. Each measuring cube can be slightly deformed in order to measure forces in one direction, and is virtually perfectly stiff with respect to the five other degrees of freedom. The boomerang's rotational velocity was kept constant by means of a servomotor, a disk with slits and a light plus photocel, and an electronic feedback system.

In the experiment the boomerang's rotation as viewed from above was clockwise (as with left-handed boomerangs), in contrast to the description in §18. The results of this experiment as they will be presented in §19 are "mirrored" in order to make them correspond to right-handed boomerangs. Measurements were done with the number of boomerang arms:

$$n = 8,4,2,0 \tag{17.1}$$

The angle of incidence ψ for the boomerang as a whole successively had the seven values:

$$\psi = -10^\circ,-5^\circ,0^\circ,5^\circ,10^\circ,15^\circ,30^\circ \tag{17.2}$$

And the reduced rotational velocity $\Omega = \omega a/V$ successively had the seven values:

$$\Omega = \tfrac{1}{2},1,1\tfrac{1}{2},2,3,5,10 \tag{17.3}$$

First, for one particular n and one particular ψ, Ω ran through the list (17.3), then the next value for ψ was selected from (17.2), and after seven values for ψ the next value for n was chosen from (17.1) The towing velocities varied between 0.44 and 1.7 m/s, and the rotational velocities between 0.54 and 4.0 revs/s. The outputs of the measuring elements were integrated over one run of about 20 seconds each time. At the lowest angular velocity this time corresponds to about 11 revolutions, which equals 22 periods for the cases with 2 arms. At the highest angular velocity 20 seconds corresponds to about 80 revolutions, which equals 640 periods for the cases with 8 arms. We may expect errors of a few percent at most in the measured average forces due to the integration time's deviating from an integral number of periods.

It is difficult to make sound estimates of the errors in the final numerical results, but it should be borne in mind that the original signals from the measuring elements were strongly oscillating. These signals were integrated and added to or subtracted from each other, which could lead to considerable errors. About half of the runs were immediately repeated; in these cases the results agreed to within a few percent. Systematic errors might result from bending of the boomerang arms under the hydrodynamic forces. The greatest axial force occurred at $\psi = 30^\circ$ for all boomerangs. The maximum average axial force per boomerang arm (F_z/n) was about 4 kgf. Momentary values may have been much greater.

§18 *Experimental boomerangs and theoretical winglet structures.*

In this section data are given concerning the profile characteristics of the boomerangs used in the experiment described in §17. Further the winglet structures simulating the experimental boomerangs are described.

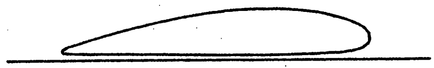


fig. 18.1 Profile of boomerang arm.

The diameter of the experimental boomerangs is 50 cm, the diameter of the central flanges 16 cm. The arms are made of dural. They have a constant profile (see fig. 18.1), with chordlength 4.03 cm and maximum thickness 0.62 cm. The boomerang arms are attached to the central flanges in such a way that their geometrical angle of attack (see §2) is:

$$\left.\begin{aligned} \alpha_o^{I} &= +.070 = +4^\circ \\ \alpha_o^{II} &= -.035 = -2^\circ \end{aligned}\right\} \quad (18.1)$$

Symbols with superscript I are valid for the normal profile, those with superscript II for the reversed profile (leading edge and trailing edge interchanged). We shall mean by the Reynolds number the quantity

$$Re \stackrel{\text{def}}{=\!=\!=} \frac{bV_e}{\nu} \qquad (18.2)$$

where b = chordlength boomerang arm, V_e = effective velocity, ν = kinematic viscosity of medium. Measurements in water at Reynolds numbers $Re = 2\times10^5$ (V = 5m/s) and $Re = 1\times10^5$ (V = 2.5m/s) gave for the profile lift coefficient C_L at small angles of incidence:

$$\frac{dC_L}{d\alpha} = 5.55 \qquad (18.3)$$

Measurements in air at $Re = 0.8\times10^5$ (V = 30m/s) gave (see §26):

$$\frac{dC_L^I}{d\alpha} \approx 5.8, \quad \frac{dC_L^{II}}{d\alpha} \approx 5.9 \tag{18.4}$$

The profile drag coefficient C_D was measured in air at Re = 0.8×10^5. For small angles of incidence

$$C_D^I \approx .025, \quad C_D^{II} \approx .05 \tag{18.5}$$

We remark that the profile characteristics are rather dependent on the Reynolds number. Air measurements at Re = 0.4×10^5 for instance, yielded a considerably lower C_L^I and a considerably higher C_D^I. A thin thread stretched in spanwise direction directly in front of the leading edge of the profile improved the characteristics, by facilitating the boundary layer's becoming turbulent (see [Schmitz, 1957]). The under-water experiments were carried out at velocities of 0.44-1.7 m/s and at 0.54-4.0 revs/s, so that the Reynolds numbers were of the order of 10^5. These circumstances correspond to those of real boomerangs in motion through air. These fly at velocities of 0-30 m/s and spin at about 10 revs/s [Hess, 1968]. Unfortunately Re $\approx 10^5$ is just the region where transitions may occur between sub- and supercritical states [Schmitz, 1957]. The situation is even more complicated, since, according to Muesmann [1958], the aerodynamic profile characteristics of rotating wings (in axial flow) may deviate from those measured on the same airfoils in straight flight. On this matter see also §26 and §33.

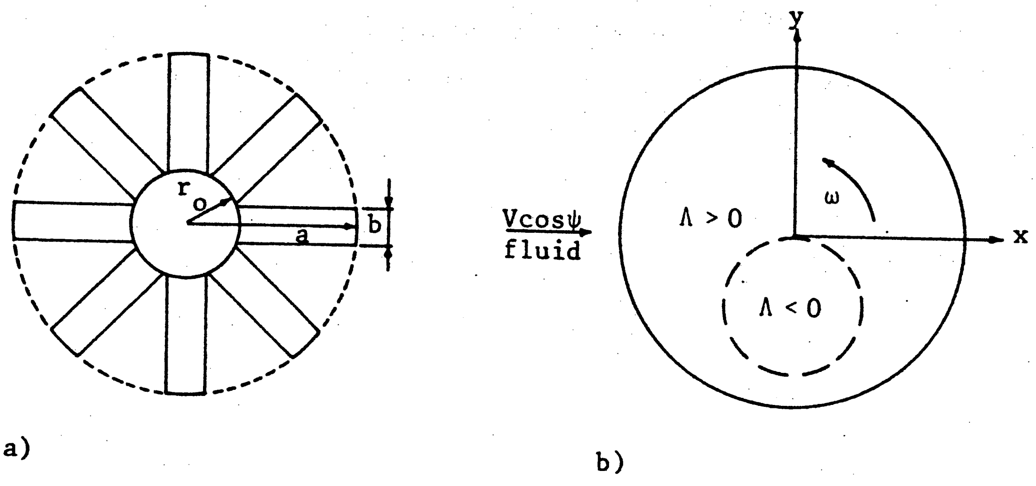


fig. 18.2 a) Boomerang and b) winglet atructure as viewed from z-direction.

We shall now deal with the winglet structures which simulate the 8-, 4- and 2-armed boomerangs used in the experiment. (See fig 18.2). Such a boomerang consists of a number of arms directed radially outward from the centre of mass. The arms lie in one plane, are equally spaced, identical, straight, and have a constant profile. They are attached to a central disk. The boomerang is placed in a homogeneous flow with velocity $\vec{V} = (V \cos\psi, 0, \sin\psi)$, and spins at the angular velocity ω counterclockwise as viewed from the positive z-direction. The region S is circular with radius a. Introduce the polar coordinates r, φ:

$$x = r \cos\varphi, \ y = r \sin\varphi \tag{18.6}$$

The same symbols are used as in §2 and §9. The structure is supposed to be kept exactly in the (x,y)-plane, hence:

$$W_z = 0 \quad \gamma = 0 \tag{18.7}$$

The velocity of the winglets is given by:

$$\vec{W} = (-\omega r \sin\varphi, \ \omega r \cos\varphi, \ 0) \tag{18.8}$$

We define:

$$\Lambda = \omega r + V \cos\psi \sin\varphi \tag{18.9}$$

The points of S for which $\Lambda = 0$ are situated on a circle through the origin with its centre at $(0, -V \cos\psi/2\omega)$. At these points the individual winglets have their leading edges changed into trailing edges and conversely. Hence in the region $\Lambda < 0$ the winglets' characteristics may differ from those in the region with $\Lambda > 0$. (They are denoted by the superscripts II and I respectively.)

For the angle β we have:

$$\beta = \varphi \,.\, \mathrm{sign}\Lambda, \quad \beta^{I} = \varphi, \quad \beta^{II} = -\varphi \tag{18.10}$$

and for the effective velocity:

$$V_e = |\Lambda| = |\omega r + V \cos\psi \sin\varphi| \tag{18.11}$$

As to the filling factor d, it depends on r only. If the original boomerang has n arms, each with constant chordlength b, then:

$$\left.\begin{aligned} b &= 4.03 \text{ cm.} \\ d(r) &= \frac{nb}{2\pi r} \qquad r_o \le r \le a \\ d(r) &= d(r_o) \qquad 0 \le r \le r_o \end{aligned}\right\} \tag{18.12}$$

where r_o is the radius of the central disk of the boomerang. Of course our theory does not apply to the central disk and its neighbourhood, but some choice for the function d has to be made for that region as well. By (18.12) d(r) is made a continuous function.

According to (2.8):

$$c(x,y) = d(x,y) \,.\, \lambda(x,y) \tag{18.13}$$

where we have put:

$$\lambda \stackrel{\text{def}}{=\!=\!=} \frac{C_L}{2\sin\alpha} \approx \frac{1}{2}\frac{dC_L}{d\alpha} \tag{18.14}$$

(Thin airfoil theory would give: $\lambda = \pi$.) Both λ and the geometrical angle of incidence α_o take two values, one for the region $\Lambda > 0$, and another for the region with $\Lambda < 0$ where the winglets' profiles experience a back to front flow:

$$\left.\begin{aligned} \lambda(r,\varphi) &= \lambda^{I} \qquad \text{if } \Lambda > 0 \\ &= \lambda^{II} \qquad \text{if } \Lambda < 0 \end{aligned}\right\} \tag{18.15}$$

$$\left.\begin{aligned} \alpha_o(r,\varphi) &= \alpha_o^{I} \qquad \text{if } \Lambda > 0 \\ &= \alpha_o^{II} \qquad \text{if } \Lambda < 0 \end{aligned}\right\} \tag{18.16}$$

According to (9.8) we have:

$$f_z(x,y) = -\mu c(r,\varphi)\left[\alpha_o + \frac{V\sin\psi - v_z}{V_e}\right]V_e^2 \tag{18.17}$$

Introducing the "reduced rotational velocity" Ω, defined by:

$$\Omega = \frac{\omega a}{V} \tag{18.18}$$

we can write (18.17) in the form:

$$-\frac{f_z}{\mu V^2} = c(r,\varphi)\left[\alpha_o(r,\varphi)(\Omega\frac{r}{a} + \cos\psi\sin\varphi)^2 + (\sin\psi + \frac{v_z}{V})\left|\Omega\frac{r}{a}\cos\psi\sin\varphi\right|\right] \qquad (18.19)$$

The radius of the boomerang is a = 25 cm, the radius of the central disk is r_o = 8 cm, hence:

$$r_o = 0.32\ a \qquad (18.20)$$

For the geometrical angle of incidence of the winglets α_o, we use (18.1). For λ we use the experimental value (18.3), hence:

$$\lambda^I = \tfrac{1}{2} \times 5.55 \qquad (18.21)$$

We remark that the region with $\lambda < 0$ is not of much importance, except perhaps in the cases with $\Omega = \frac{1}{2}$, since V_e is small there anyway. Therefore the profile characteristics of the reversed profile are not too important, and since they were not measured in water, we simply take $\lambda^{II} = \lambda^I$. The winglet structures are now completely determined.

§19 *Comparison between experiment and theory.*

In this section the results of the experiment described in §17 are compared with the theoretical results. The theoretical calculations are based on the winglet structures described in the preceding section. The components F_{oz}, F_{ox} and T_{oz} are computed according to the theory of chapters I, II, III, for each of the seven values of ψ from (17.2) and for each of the seven values for Ω from (17.3). Both program B ($\psi = 0$) and program CB ($\psi \neq 0$) are used with $N = 6$, $M = 6$ and TOL. = .02. According to §15 the relative inaccuracies may amount to a few percent.

The experimentally determined values for F_{oz}, F_{ox} and T_{oz} are corrected by subtracting the corresponding values measured without arms, and plotted as functions of the angle ψ, with Ω as a parameter. Where a measurement was repeated, the outcomes are averaged, and this average is plotted. The theoretical results are plotted in the form of continuous curves drawn by hand through the computed points. Figures 19.1, 19.2, 19.3 represent the axial force F_{oz} for 8, 4 and 2 arms respectively, and figures 19.4, 19.5 and 19.6 represent the axial torque T_{oz}. Plots for the component F_{ox} are not included here, but can be found in [Hess, 1973, §27].

Let us first consider the results for the axial force component F_{oz} (figures 19.1, 19.2, 19.3). The agreement between theory and experiment appears to be reasonable, even, somewhat surprisingly, for the case with only 2 arms. The deviations are greatest where they could be expected:
1º For $\Omega = 10$, the induced velocities are so large that a linear theory cannot be justified (heavy loading).
2º For $\psi = 30^\circ$ and low values for Ω, the angles of incidence of the boomerang arms become so large that these wings may be stalled. We would therefore expect the experimental values of F_{oz} to drop below the theoretical values. This tendency is generally confirmed by the graphs.
The dip in the theoretical curves for high values of Ω at $\psi = 0^\circ$ must be due to the vorticity passing the region S at very close distance according to the theory. The experimental points seem to have a corresponding dip, more spread out and at $\psi \approx 5^\circ - 10^\circ$.

As far as the components parallel to the (x,y)-plane: F_{ox}, F_{oy}, T_{oz} are concerned, our theory cannot be expected to yield realistic results. For one thing, viscosity is neglected completely. The theoretical values for F_{ox} and T_{oz} do not resemble the experimental values at all. We can, however, take the drag other than induced drag into account in a simple way. According to measurements in air, the profile drag coefficient of the boomerang arms is given by (18.5). For simplicity we take C_D^{II} equal to C_D^I (the region with $\Lambda < 0$ is not of much importance) and choose

$$C_D^{II} = C_D^I = .025 \tag{19.1}$$

as the profile drag coefficient. It is quite simple to incorporate the additional force f_1 (see §2) due to this drag into the computing programs B and CB: at each pivotal point we have the additional contribution:

$$f_1 = -\tfrac{1}{2}\,\mu d V_e^2 C_D \tag{19.2}$$

The effect of this addition is that the theoretical curves in the graphs for F_{ox} and T_{oz} vs . ψ are shifted up or down, for each value of Ω by a different amount. This modification was incorporated in the theoretical graphs (figs. 19.4, 19.5, 19.6) for the axial torque T_{oz}. There appears to be only a poor quantitative agreement between theory and experiment, although qualitatively the agreement is not too bad. In particular there is an almost perfect correspondence between theory and experiment as to the sign of T_{oz}. A negative T_{oz} means that the boomerang's rotation is slowed down by the hydrodynamic forces, a positive T_{oz} means autorotation. Like the results for F_{oz}, the deviations between theory and experiment are greatest for $\Omega = 10$ and for $\psi = 30°$.

As regards component F_{ox} the agreement between theory and measurements is somewhat less than for T_{oz}. Graphs are given in [Hess, 1973, §27].

We may draw the following conclusions. The results presented in this section suggest that the present theory yields reasonable values for the average forces on systems consisting of a number of rotating wings, at least as far as the axial components F_{oz} and T_{oz} are concerned.

Serious deviations from theory may occur, 1° if the systems are heavily loaded, and 2° if the wings are stalled. The latter shortcoming of the present theory will be remedied by a modification described in §21, where boomerang arms with non-linear profile characteristics are dealt with.

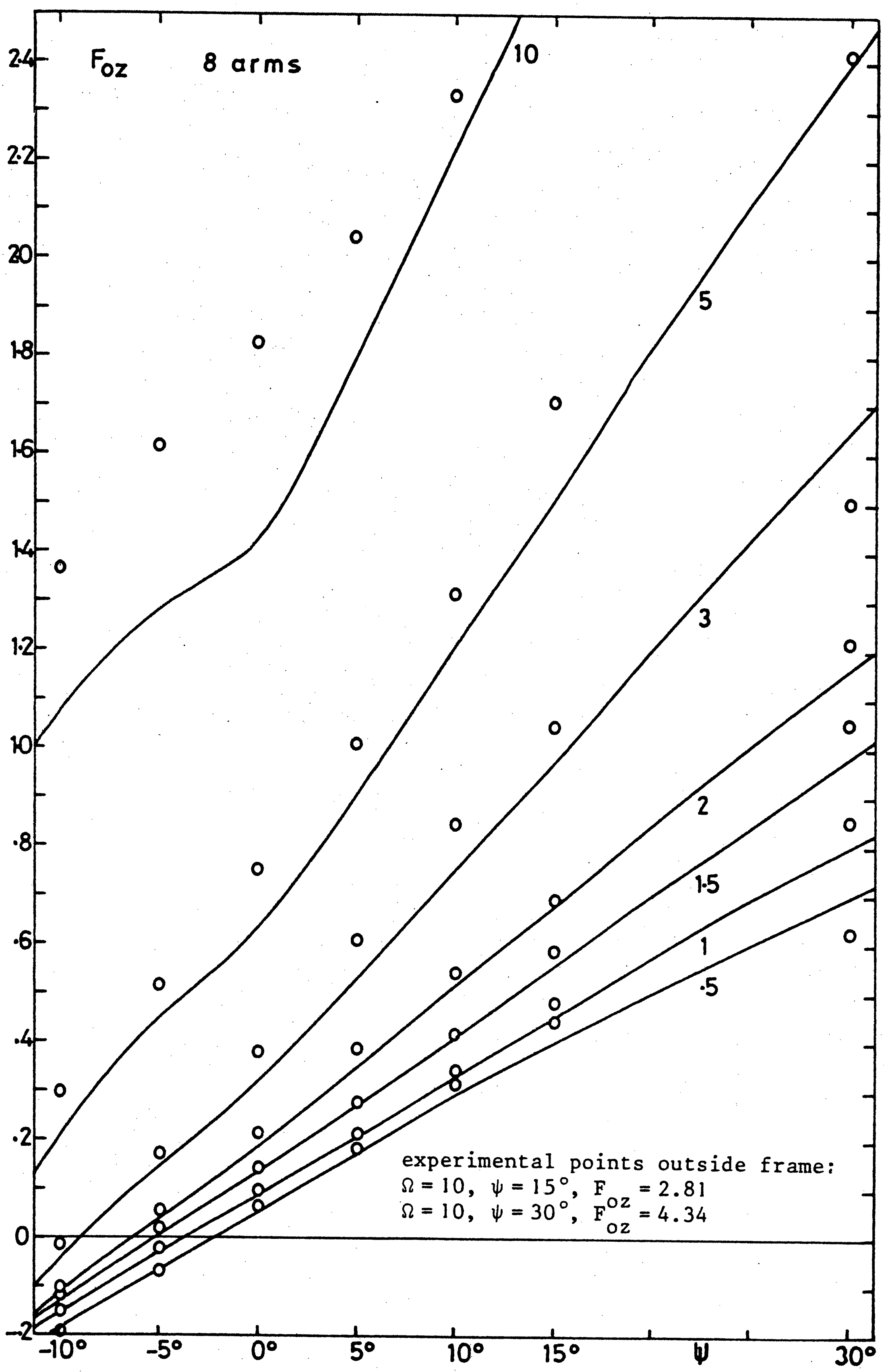


fig. 19.1. Axial force F_{oz} for 8 arms. Theory: ———— Experiments: o

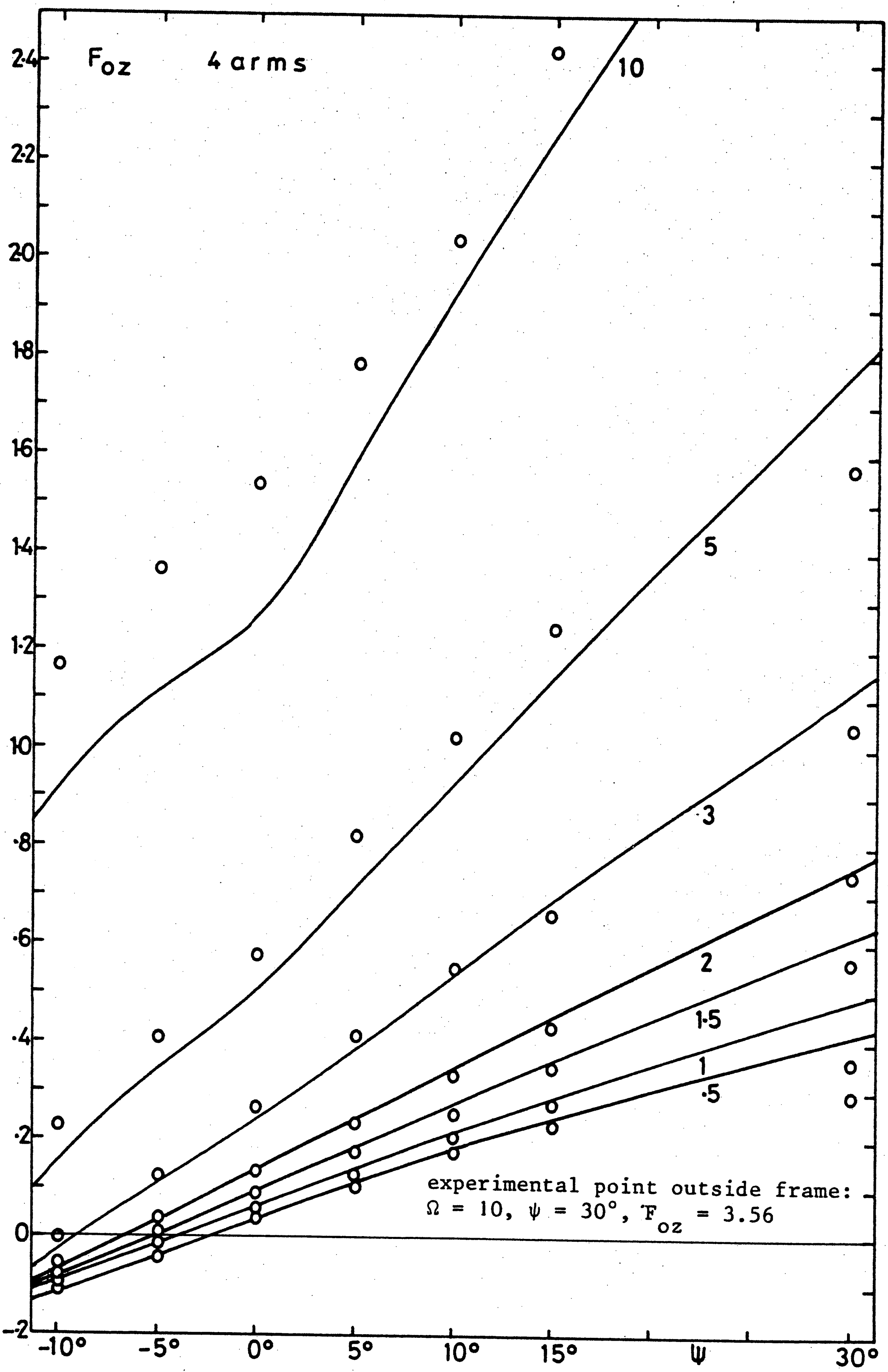


fig. 19.2. Axial force F_{oz} for 4 arms. Theory: ——— Experiments: o

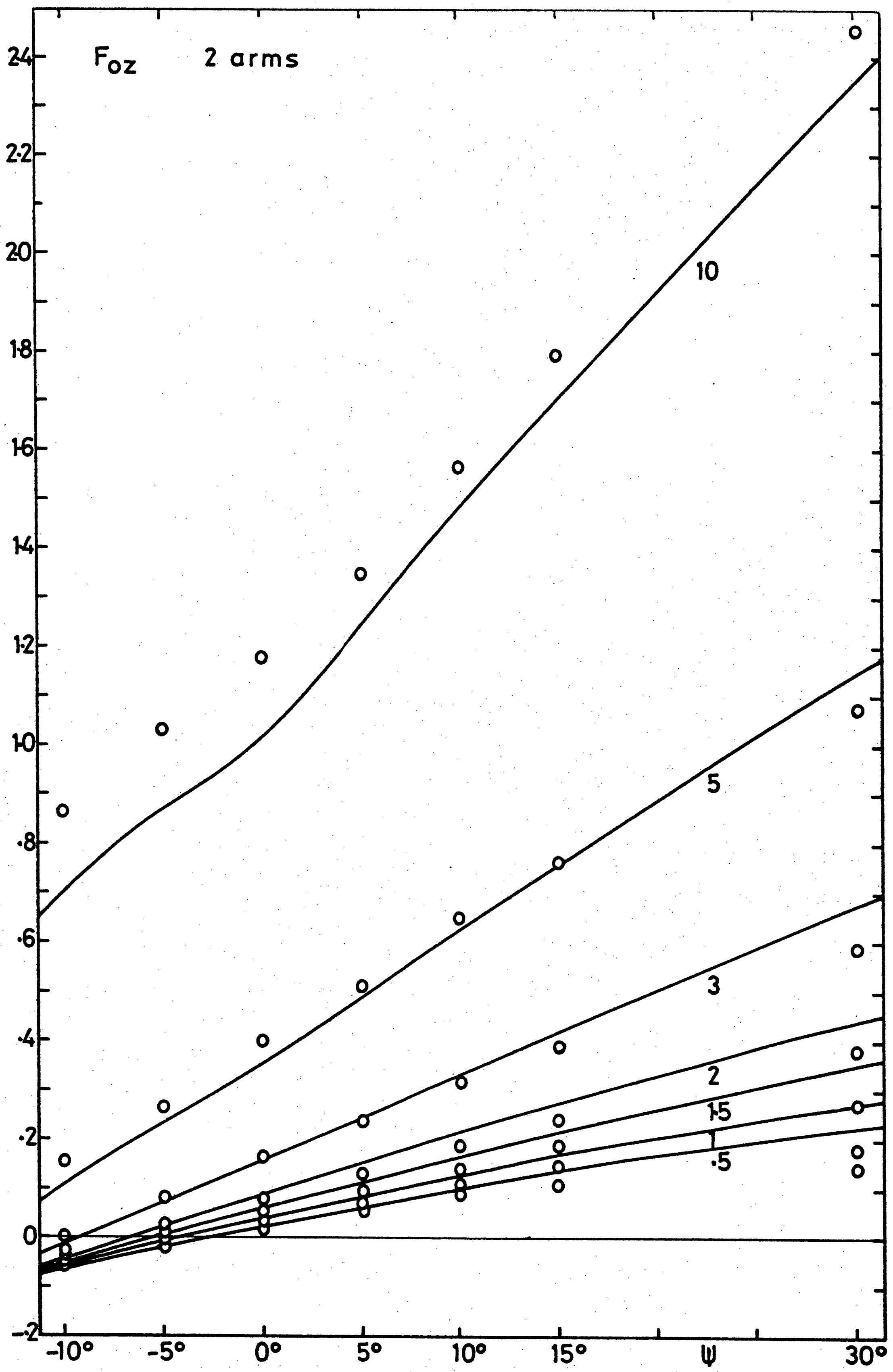


fig. 19.3. Axial force F_{oz} for 2 arms. Theory: —— Experiments: o

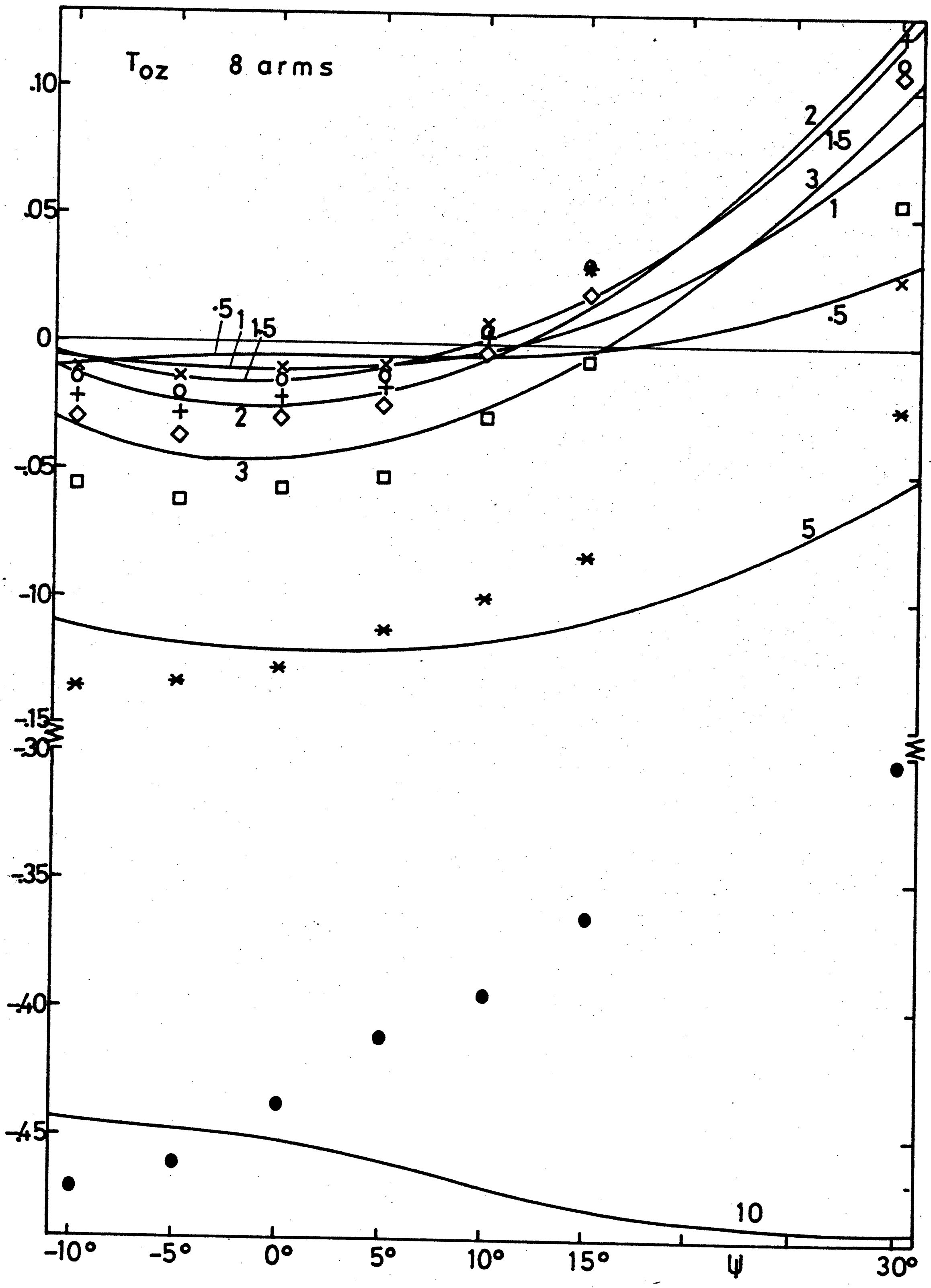


fig. 19.4. Axial torque T_{oz} for 8 arms. Theory: ——————
Measurements: × .5, ○ 1, + 1.5, ◇ 2, □ 3, ✱ 5, ● 10 (Ω values).

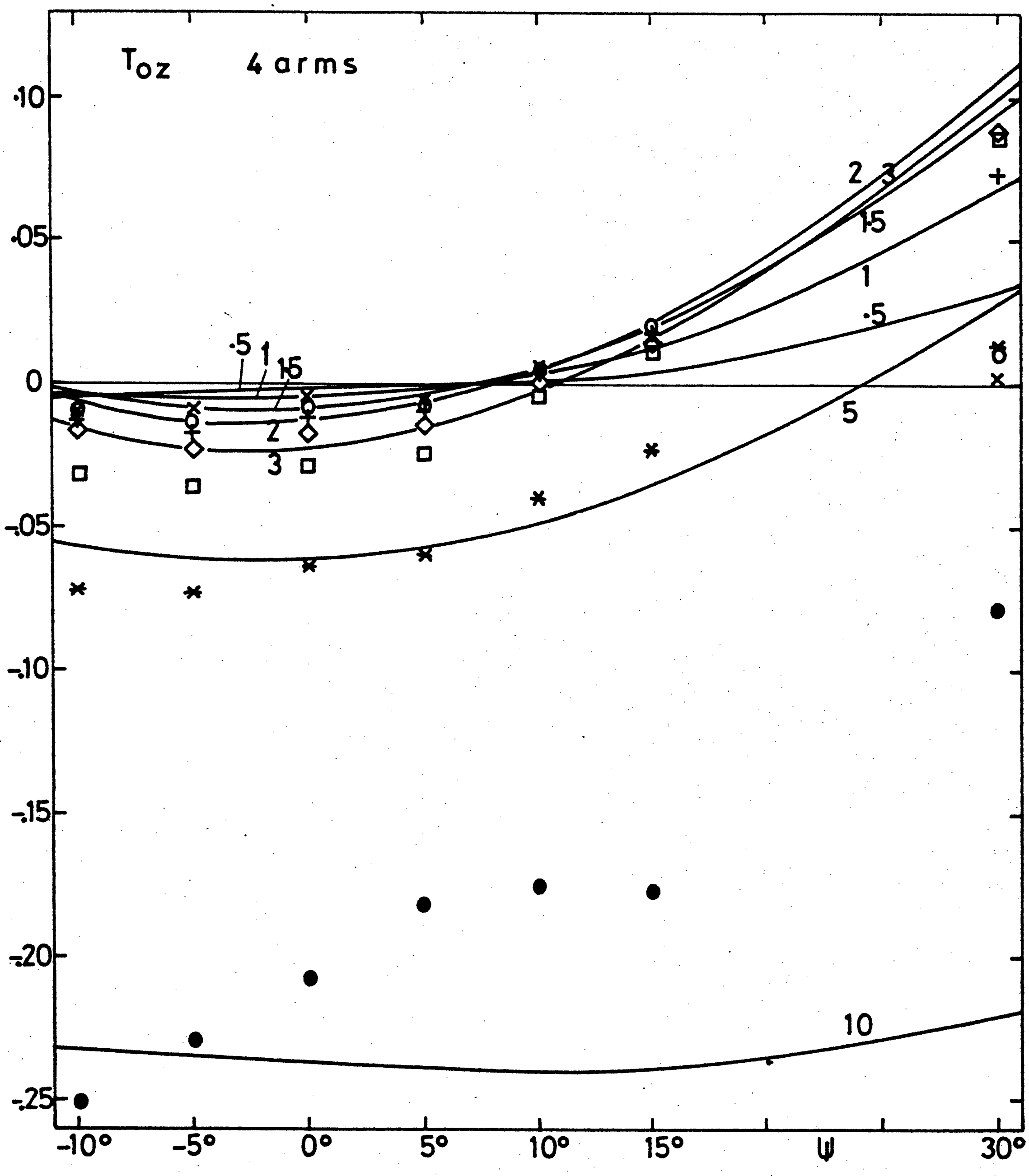


fig. 19.5. Axial torque T_{oz} for 4 arms. Theory: ————
Measurements: × .5, ○ 1, + 1.5, ◇ 2, □ 3, ⁎ 5, ● 10 (Ω values).

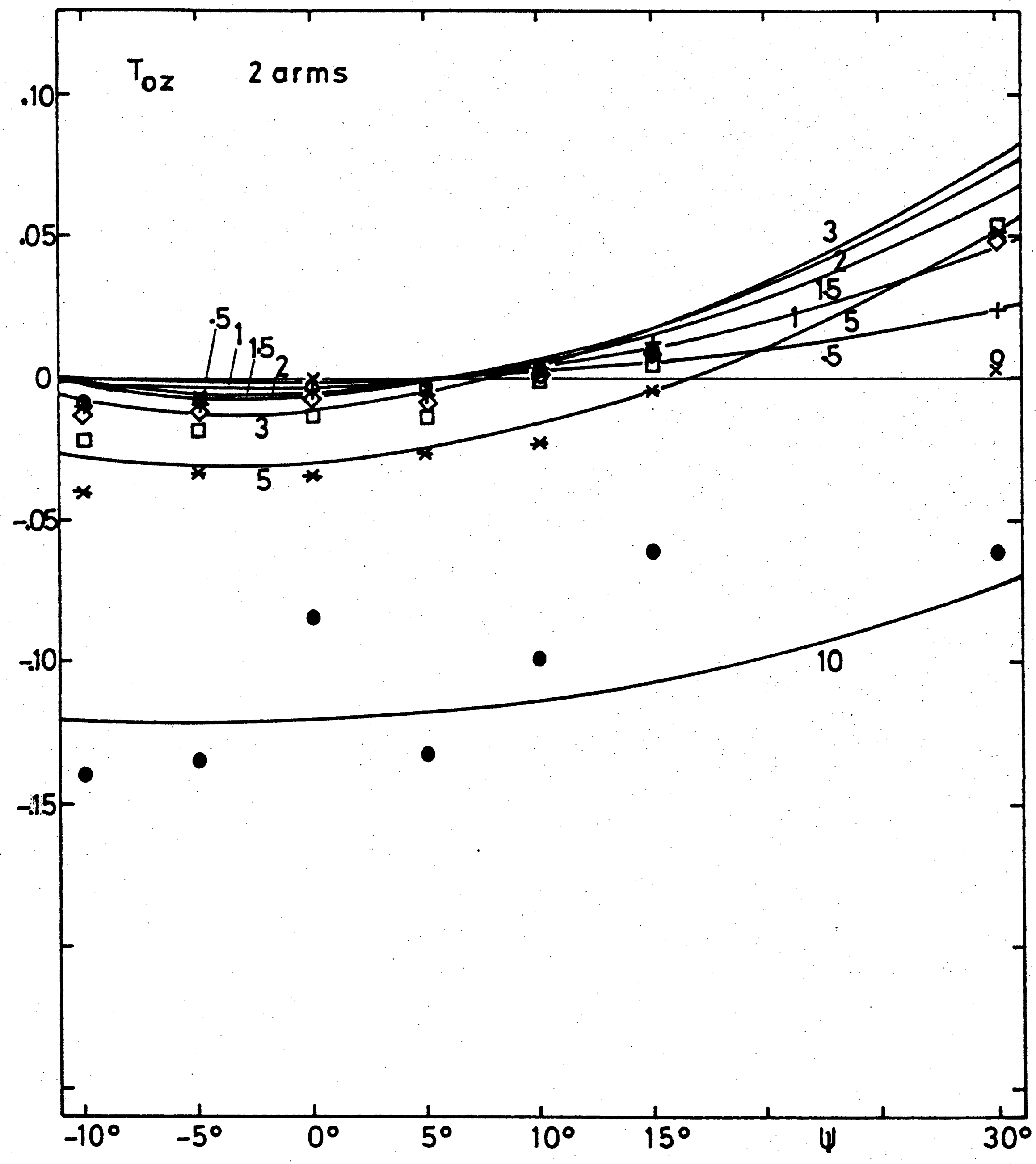


fig. 19.6. Axial torque T_{oz} for 2 arms. Theory: ________
Measurements: × .5, ○ 1, + 1.5, ◇ 2, □ 3, ✳ 5, ● 10 (Ω values).

CHAPTER V

SOME MODIFICATIONS OF THE WINGLET MODEL.

§20 *Superimposed winglet systems.*

This chapter deals with some modifications of the winglet model developed in order to make the model better adapted to the conditions pertaining to real boomerangs.

The numerical and experimental examples considered in chapter IV were exclusively concerned with systems consisting of a number of *identical* airfoils. In the theoretical winglet model of chapters I and II these airfoils were "smeared out" to one winglet system. The parameters characterizing such a winglet system: d, α_o, β, γ, $\vec{W}$ (see §2) were single-valued functions of x and y. Systems with rotating airfoils of different shapes cannot be simulated by these winglet systems. A conventional boomerang, for example, has two arms which are not directed radially outward from the axis of rotation: one arm "precedes", the other arm "follows" this axis [Hess, 1968] (see also fig. 20.1). Hence each arm gives rise to a different $\beta(x,y)$. However, only a relatively simple modification is required in order to extend the winglet model to systems with unequal airfoils. Each of the airfoils can be individually "smeared out" to a winglet system, so that the system as a whole is replaced by two or more superimposed winglet systems, each covering the same region S in the (x,y)-plane. This section describes how these superimposed winglet systems can be simultaneously treated by the methods of chapters I, II, III.

Suppose that the original system under consideration consists of n (possibly unequal) airfoils. We denote the characteristic parameters belonging to each airfoil by a superscript (j), $j = 1...n$. Such an airfoil can be smeared out to a winglet system characterized by the functions

$$d^{(j)}(x,y), \alpha_o^{(j)}(x,y), \beta^{(j)}(x,y), \gamma^{(j)}(x,y), \vec{W}^{(j)}(x,y), \quad j = 1...n$$

according to §2. The force per unit of area of S exerted by each winglet system on the fluid is $\vec{f}^{(j)}(x,y)$, $j = 1\ldots n$. The total force per unit of area is then given by

$$\vec{f}(x,y) = \sum_{j=1}^{n} \vec{f}^{(j)}(x,y) \qquad . \tag{20.1}$$

The contents of §2 and §9 (in which winglet structures were described for the cases $\psi = 0$ and $\psi \neq 0$ respectively) have to be only slightly modified. For the case $\psi \neq 0$ we have instead of (9.5):

$$V_e^{(j)} = (V\cos\psi - W_x^{(j)})\sin\beta^{(j)} + W_y^{(j)}\cos\beta^{(j)}, \qquad j = 1\ldots n \tag{20.2}$$

and instead of α_e in (9.6):

$$\alpha_e^{(j)} = \frac{-[(V\cos\psi - W_x^{(j)})\cos\beta^{(j)} - W_y^{(j)}\sin\beta^{(j)}]\gamma^{(j)} - W_z^{(j)}}{V_e^{(j)}}, \; j = 1\ldots n \tag{20.3}$$

For the case $\psi = 0$ we obtain the modified versions of (2.14) and (2.15) by taking $\psi = 0$ in (20.2) and (20.3) respectively. The formulas (2.16) through (2.19), describing winglet systems for $\psi = 0$, are also still valid for each of the n winglet systems individually. The same is true for the formulas (9.7) and (9.8) in the case $\psi \neq 0$. In §4 the general character of the integral equation for $\psi = 0$ was considered. We remark that in the present case the integral equation can still be written in the form (4.1), provided that L, P, Q are not defined by (4.2) but by:

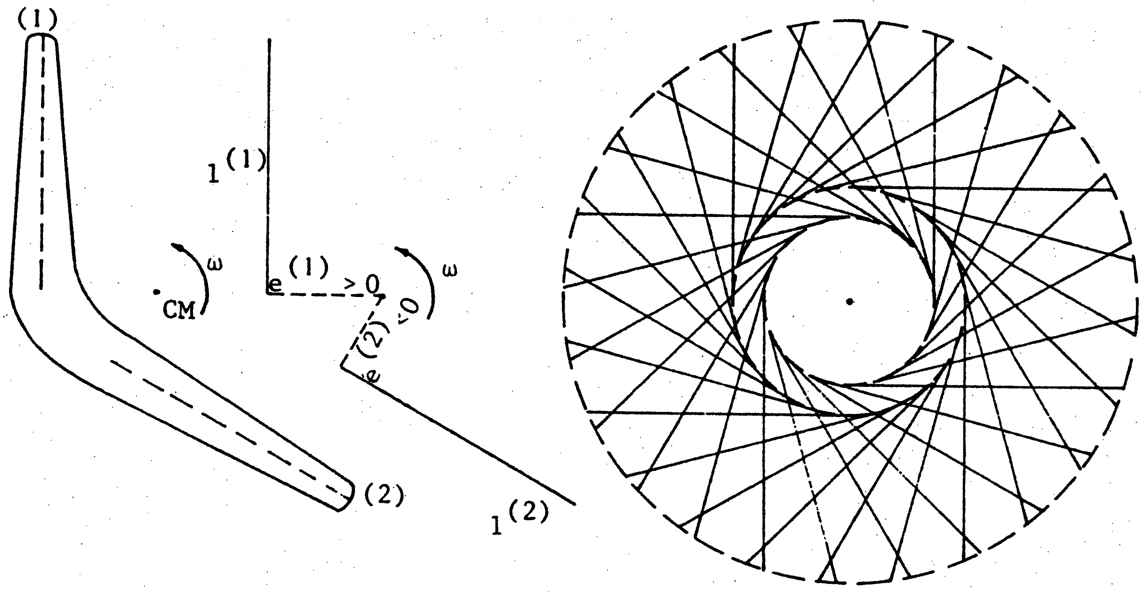
$$\left.\begin{aligned} L &= \frac{-f_z}{\mu V^2} = \sum_{j=1}^{n} \frac{-f_z^{(j)}}{\mu V^2} \\ P &= \sum_{j=1}^{n} c^{(j)}(\alpha_o^{(j)} + \alpha_e^{(j)})\frac{V_e^{(j)^2}}{V^2} \\ Q &= \sum_{j=1}^{n} c^{(j)} \frac{V_e^{(j)}}{V} \end{aligned}\right\} \tag{20.4}$$

§9, for the case $\psi \neq 0$, can be modified in a completely analogous way.

The solution of the integral equation proceeds as follows. As before, the elementary induced velocities $v_{pl}(X_\mu, y_\nu)$ are computed according to the methods outlined in the chapters I, II and III. The coefficients a_{pl}, $(p = 0\ldots M-1,\ l = 1\ldots N)$ are found by the collocation method. From the a_{pl} follow the values for the three components F_{oz}, T_{ox}, T_{oy} according to §7. Next the values for the induced velocity at the pivotal points $v_z(X_\mu,y_\nu)$, $(\mu = 1\ldots M,\ \nu = 1\ldots N)$ are calculated. Then (2.18) and (2.19) or (9.7) and (9.8) yield values for $f_x^{(j)}(X_\mu,y_\nu)$, $f_y^{(j)}(X_\mu,y_\nu)$, $f_z^{(j)}(X_\mu,y_\nu)$, $(\mu = 1\ldots M,\ \nu = 1\ldots N,\ j = 1\ldots n)$. Hence follow $f_x(X_\mu,y_\nu)$, $f_y(X_\mu,y_\nu)$, $f_z(X_\mu,y_\nu)$, from which the coefficients b_{pl}, c_{pl}, $(p = 0\ldots M-1,\ l = 1\ldots N)$ are computed, according to §7. Finally values are obtained for F_{ox}, F_{oy}, T_{oz}. This process is rendered schematically as follows:

$$\begin{tikzcd}
 & \boxed{a_{pl}} \arrow[dl] \arrow[d] \arrow[dr] & \\
\boxed{f_z(X_\mu,y_\nu)} & \boxed{v_z(X_\mu,y_\nu)} \arrow[d] & \boxed{F_{oz},\ T_{ox},\ T_{oy}} \\
 & \boxed{f_x^{(j)}(X_\mu,y_\nu),\ f_y^{(j)}(X_\mu,y_\nu),\ f_z^{(j)}(X_\mu,y_\nu)} \arrow[d] & \\
 & \boxed{f_x(X_\mu,y_\nu),\ f_y(X_\mu,y_\nu),\ f_z(X_\mu,y_\nu)} \arrow[d] & \\
 & \boxed{b_{pl},c_{pl}} \arrow[d] & \\
 & \boxed{F_{ox},\ F_{oy},\ T_{oz}} &
\end{tikzcd} \qquad (20.5)$$

The concept of superimposed winglet systems is developed in order to adapt the methods of chapters I and II to boomerangs of the conventional type in particular. We shall now describe in what manner such a boomerang can be reduced to two superimposed winglet systems. Fig. 20.1(a) gives a sketch of a conventional boomerang consisting of two arms or airfoils. The boomerang's rotation in the figure is counterclockwise. The axis of rotation passes through the boomerang's centre of mass, denoted by CM. Each arm is considered to be situated in the neighbourhood of a straight line. Arm number 1 "precedes" the axis of rotation, it has a positive excentricity $e^{(1)}$, while arm number 2 "follows" the axis of rotation and has a negative excentricity $e^{(2)}$. The excentricity of an arm is the distance between the axis of rotation and the straight reference line belonging to the arm, see fig. 20.1(b).



a) b) c)

fig. 20.1 Sketch of conventional boomerang and its winglet systems.

Winglet system 1 or 2 is obtained by smearing out straight line 1 or 2 over one complete period of rotation. Each straight line is considered to begin at the point nearest to the axis of rotation, and to end at the wing tip. Thus the airfoils are considered to have lengths $l^{(1)}$ and $l^{(2)}$ respectively. The central part, or "elbow", of the boomerang probably is not very well simulated this way, but this problem appears to be very difficult to solve anyhow. If both winglet systems should have the same radius, the arm lengths and excentricities should satisfy the condition:

$$l^{(1)^2} + e^{(1)^2} = l^{(2)^2} + e^{(2)^2} = a^2 \qquad (20.6)$$

If (20.6) is not satisfied, the arm lengths $l^{(j)}$ are modified as follows. The radius a of the system is now defined by:

$$a^2 = \frac{1}{n} \sum_{j=1}^{n} a^{(j)^2}, \quad a^{(j)^2} = l^{(j)^2} + e^{(j)^2} \qquad (20.7)$$

And the modified arm length $\tilde{l}^{(j)}$ by:

$$\tilde{l}^{(j)} = \sqrt{a^2 - e^{(j)^2}} \qquad (20.8)$$

To compensate for the change in arm length, the chord length at the tip of the arm is modified in such a way that the area of the arm remains unchanged. If the arm has a trapezium planform with chord length $c_t^{(j)}$ at the tip and $c_r^{(j)}$ at the root, then the modified chord length at the tip is:

$$\tilde{c}_t^{(j)} = \frac{l^{(j)}}{\tilde{l}^{(j)}} (c_t^{(j)} + c_r^{(j)}) - c_r^{(j)} \qquad (20.9)$$

Of course, this procedure can only be reasonably applied to cases in which the $a^{(j)}$ have relatively small differences. As to the empty inner circles within S: $r < e^{(1)}$ and $r < e^{(2)}$ respectively: the situation here cannot be accounted for by our winglet model.

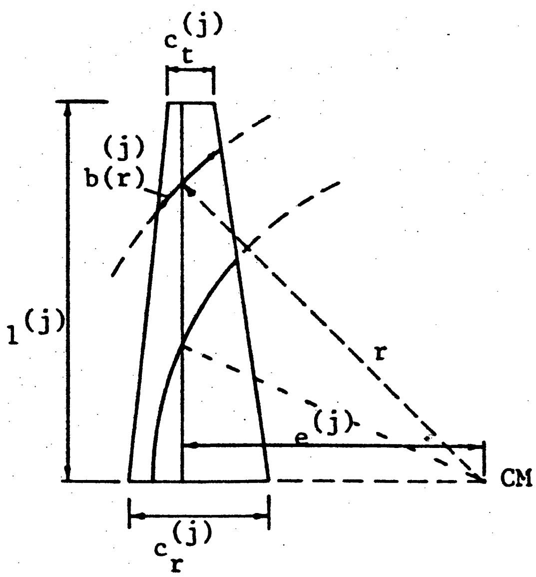


fig. 20.2 Determination of "density function" $d^{(j)}(x,y)$.

The "density function" or "filling factor" $d^{(j)}(x,y)$ belonging to winglet system number j (j = 1,2 for conventional boomerangs) can be determined as follows. A circle with the axis of rotation as centre and radius r passes over arm j for a length of, say, $b^{(j)}(r)$. (In figure 20.2 this length is indicated by a heavy drawn line for two different values of r.) We now put:

$$d^{(j)}(x,y) = d^{(j)}(r) = \frac{b^{(j)}(r)}{2\pi r} \tag{20.10}$$

The other parameters characterizing winglet system j: $\alpha_o^{(j)}(x,y)$, $\beta^{(j)}(x,y)$, $\gamma^{(j)}(x,y)$, $\vec{W}^{(j)}(x,y)$ follow from the shape and motion of airfoil j in an obvious way.

§21 *Non-linear profile characteristics.*

Up to this point we have dealt with completely linearized systems. On the one hand f_z depended linearly on v_z according to (2.19) and (9.8), on the other hand v_z depended linearly on f_z according to (3.17) and (8.11) or (8.12). The resulting integral equations were linear. As it turns out, real boomerangs may operate under such conditions that their airfoils have rather large angles of attack, for instance if $\psi \gtrsim 10^\circ$ and $\Omega \approx 1$. The profile characteristics of the airfoils in these circumstances may strongly deviate from linearity, and stall may occur. In this section we shall describe a modification of the linear winglet model which allows arbitrary lift and drag profile characteristics of airfoils to be taken into account. We shall concentrate on the case $\psi \neq 0$, the case $\psi = 0$ differs only in trivialities.

Suppose that for each of the n airfoils of the original system a winglet system has been constructed as described in §20. According to (2.4) the angle of attack $\alpha^{*(j)}$, (j = 1...n) is given by:

$$\alpha^{*(j)}(x,y) = \operatorname{arctg} \frac{v_{r3}^{(j)}(x,y)}{v_{r1}^{(j)}(x,y)}, \qquad j = 1...n \tag{21.1}$$

Hence

$$\left.\begin{aligned} \cos\alpha^{*(j)} &= v_{r1}^{(j)} / \sqrt{v_{r1}^{(j)2} + v_{r3}^{(j)2}} \\ \sin\alpha^{*(j)} &= v_{r3}^{(j)} / \sqrt{v_{r1}^{(j)2} + v_{r3}^{(j)2}} \end{aligned}\right\} j = 1...n \tag{21.2}$$

The profile lift and drag coefficients, respectively denoted by $C_L^{(j)}$ and $C_D^{(j)}$, may be arbitrary functions of $\alpha_0^{(j)} + \alpha^{*(j)}$. We then have for the local lift $-f_L^{(j)}$, respectively the local drag $-f_D^{(j)}$, per unit of area:

$$\left.\begin{aligned} -f_L^{(j)}(x,y) &= \tfrac{1}{2}\mu\, d^{(j)} C_L^{(j)} [v_{r1}^{(j)2} + v_{r3}^{(j)2}] \\ -f_D^{(j)}(x,y) &= \tfrac{1}{2}\mu\, d^{(j)} C_D^{(j)} [v_{r1}^{(j)2} + v_{r3}^{(j)2}] \end{aligned}\right\} j = 1...n \tag{21.3}$$

$f_L^{(j)}$ is exerted by the winglets on the fluid in the direction $(V_{r3}^{(j)},0,-V_{r1}^{(j)})$, $f_D^{(j)}$ in the direction $(-V_{r1}^{(j)},0,-V_{r3}^{(j)})$. Thus, conditions B and C of §2 are modified. From (21.3) follows, instead of (2.11):

$$\left.\begin{aligned} f_1^{(j)} &= f_D^{(j)} \cos\alpha^{*(j)} - f_L^{(j)} \sin\alpha^{*(j)} \\ f_2^{(j)} &= 0 \\ f_3^{(j)} &= f_L^{(j)} \cos\alpha^{*(j)} + f_D^{(j)} \sin\alpha^{*(j)} \end{aligned}\right\} j = 1\ldots n \qquad (21.4)$$

These components can be transformed to $f_x^{(j)}$, $f_y^{(j)}$, $f_z^{(j)}$ according to (2.1). Instead of (2.19) and (9.8) we now obtain:

$$f_z^{(j)} = -\tfrac{1}{2}\mu\, c^{(j)} \sqrt{V_{r1}^{(j)2} + V_{r3}^{(j)2}} \cdot [V_{r1}^{(j)} C_L^{(j)} + V_{r3}^{(j)} C_D^{(j)}], \; j = 1\ldots n \qquad (21.5)$$

In this general case $f_z = \sum_{j=1}^{n} f_z^{(j)}$ does not depend on the induced velocity v_z in a linear way anymore.

On the other hand we wish to retain the linear relationship (8.11) for $\psi \neq 0$ and (3.17) for $\psi = 0$ between v_z and f_z. A non-linear fluid dynamic theory would be something completely different, and extremely difficult to set up anyway. Just like in §9 we must suppose 1$^{\underline{o}}$ that the induced velocity $\vec{v}$ is small compared to V, 2$^{\underline{o}}$ that the field of forces $\vec{f}$ has only relatively small x and y components, and 3$^{\underline{o}}$ that the only coupling between force field and induced velocity field is between the z-components of both. This coupling is described by the linear expression (8.11) on the one hand and by (21.5), the non-linear modification of (9.8), on the other hand. The main difference from §9 is that condition (9.4), which can be written as

$$\left.\begin{aligned} & V_{r3}^{(j)} \ll V_{r1}^{(j)} \\ \text{or} \qquad & \\ & \frac{V\sin\psi}{V_e^{(j)}} \ll 1 \end{aligned}\right\} j = 1\ldots n \qquad (21.6)$$

is not satisfied. On the contrary, the theory of this chapter is aimed just at cases in which (21.6) is *not* satisfied, and $a_e^{(j)}$ and hence $a^{*(j)}$ may be large angles.

faces. However, with our pervious lifting surfaces the fluid may behave linearly because of the lifting surface's being only lightly loaded, while the winglets nevertheless have non-linear characteristics. In terms of porous lifting surfaces: if the leaking velocity through the surface is increased, the pressure difference between both sides of it may not increase proportionally; it may for instance reach a maximum and then stay constant or even decrease. It is clear that the porosity coefficient (see §4) of such a porous surface is not a constant anymore. Of course, the airfoils of the original system, which is supposed to be simulated by the winglet model, can be heavily loaded individually. (For real boomerangs they may have lift coefficients in the neighbourhood of 1.5, see §32.) It must be admitted, however, that by now the winglet model appears to have been stretched to the utmost limits of its applicability. It would be difficult to theoretically justify the application of either the strictly linear theory, or the present modified theory, to boomerangs having only two arms and rotating at $\omega a/V \approx 1$.

The non-linear integral equation formed by (21.5) and (8.11) is solved by an iteration method. In order to obtain a first approximation we start by setting the profile drag coefficients equal to zero and taking lift coefficients which depend linearly on the angles of attack, just like in §9. These lift coefficients are chosen in such a way that they reasonably approximate the true values for small angles of attack. According to the methods of chapters I and II we then obtain a first estimate for the coefficients a_{pl}, ($p = 0 \ldots M-1$, $l = 1 \ldots N$). Next the values of the induced velocity at the pivotal points $v_z(X_\mu, y_\nu)$, ($\mu = 1 \ldots M$, $\nu = 1 \ldots N$) are calculated. Then we obtain values for $f_x^{(j)}(X_\mu, y_\nu)$, $f_y^{(j)}(X_\mu, y_\nu)$, $f_z^{(j)}(X_\mu, y_\mu)$, ($\mu = 1 \ldots M$, $\nu = 1 \ldots N$, $j = 1 \ldots n$) according to (21.4) and (2.1). From these follow directly $f_x(X_\mu, y_\nu)$, $f_y(X_\mu, y_\nu)$, $f_z(X_\mu, y_\nu)$. According to §7 values are computed for the coefficients b_{pl}, c_{pl}, ($p = 0 \ldots M-1$, $l = 1 \ldots N$) and, by the same method, also new values for the a_{pl}. Hence follow estimates for the six resulting components F_{oz}, T_{ox}, T_{oy}, F_{ox}, F_{oy}, T_{oz} according to §7. The new values for the a_{pl} are now used instead of the old ones and the process is repeated until a certain criterion is satisfied. For instance, the differences between the newest estimates and the previous estimates for the a_{pl} or for the six resulting components must be smaller than a certain preset value. The process is rendered schematically as follows:

$$
\begin{array}{c}
\boxed{a_{pl}} \\
\downarrow \\
\boxed{v_z(X_\mu, y_\nu)} \\
\downarrow \\
\boxed{f_x^{(j)}(X_\mu, y_\nu),\ f_y^{(j)}(X_\mu, y_\nu),\ f_z^{(j)}(X_\mu, y_\nu)} \\
\downarrow \\
\boxed{f_x(X_\mu, y_\nu),\ f_y(X_\mu, y_\nu),\ f_z(X_\mu, y_\nu)} \\
\downarrow \\
\boxed{a_{pl},\ b_{pl},\ c_{pl}} \\
\downarrow \\
\boxed{\begin{array}{c} F_{oz},\ T_{ox},\ T_{oy} \\ F_{ox},\ T_{oy},\ T_{oz} \end{array}}
\end{array}
\qquad (21.7)
$$

if convergence criterion is not satisfied

A fundamental weakness of this iteration method is, of course, the possibility that the process does not converge. Numerical calculations showed that indeed a lack of convergence occasionally occurs: if the convergence criterium is set too sharp, the solution for the load function f_z may tend to undulate with increasing amplitude.

§22 *An example.*

In the example considered here the modifications of §20 and §21 are applied. The theoretical boomerang described in this section is meant to be a simulation of an actual boomerang (L1) used in windtunnel experiments (see §28).

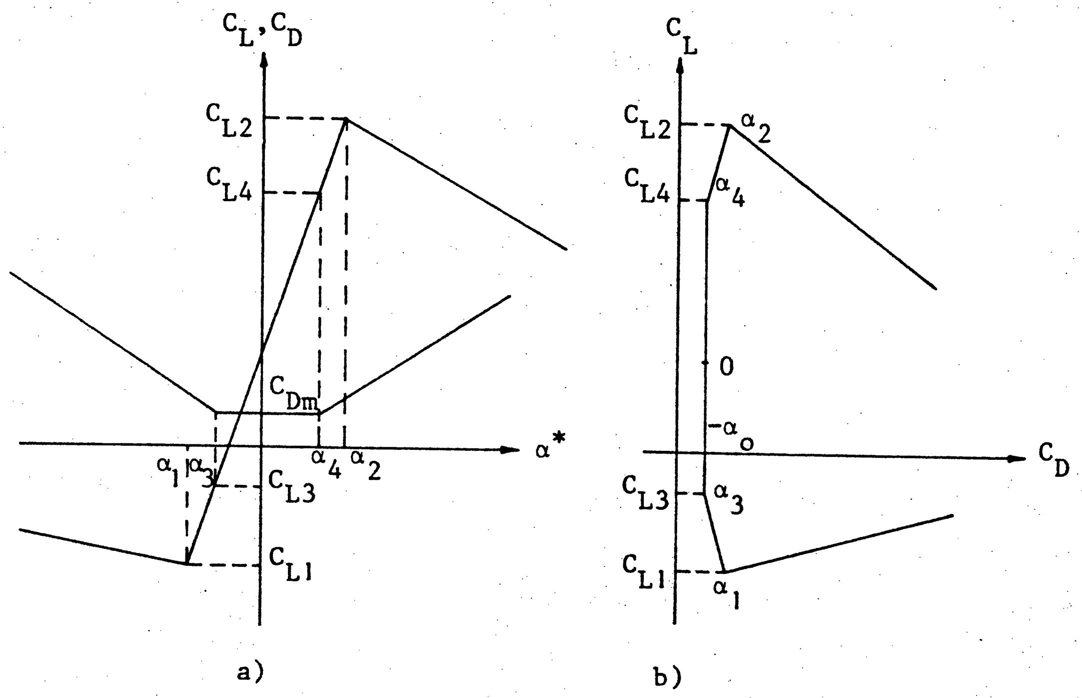


fig. 22.1 (a) C_L and C_D vs α^*, (b) polar curve.

The lift and drag coefficients of the model boomerang arms as functions of the local angles of incidence should be reasonably realistic. On the other hand, they should be as little complicated as possible. Our choice is indicated in fig. 22.1, which provides a schematic picture. α^* is the angle by which the airflow relative to the local profile minus its spanwise component deviates from the (x,y)-plane. The superscripts (j), indicating the association with a certain airfoil or arm number j ($j = 1,2$), are omitted. The profile lift coefficient C_L and the profile drag coefficient C_D are determined by:

$$\left.\begin{aligned}
C_L &= (\alpha^* + \alpha_o) \cdot dC_L && \text{if } \alpha_1 \leq \alpha^* \leq \alpha_2 \;(C_{L1} \leq C_L \leq C_{L2}) \\
C_L &= C_{L2} \cdot \frac{\frac{1}{2}\pi-\alpha^*}{\frac{1}{2}\pi-\alpha_2} && \text{if } \alpha_2 < \alpha^* \leq \tfrac{1}{2}\pi \\
C_L &= C_{L1} \cdot \frac{\frac{1}{2}\pi+\alpha^*}{\frac{1}{2}\pi+\alpha_1} && \text{if } -\tfrac{1}{2}\pi \leq \alpha^* < \alpha_1
\end{aligned}\right\} \quad (22.1)$$

and

$$\left.\begin{aligned}
C_D &= C_{Dm} && \text{if } \alpha_3 \leq \alpha^* \leq \alpha_4 \\
C_D &= C_{Dm} + (\alpha^*-\alpha_4) \cdot dC_D && \text{if } \alpha_4 < \alpha^* \leq \tfrac{1}{2}\pi \\
C_D &= C_{Dm} + (\alpha_3-\alpha^*) \cdot dC_D && \text{if } -\tfrac{1}{2}\pi \leq \alpha^* < \alpha_3
\end{aligned}\right\} \quad (22.2)$$

dC_L and dC_D stand for $dC_L/d\alpha$ and $dC_D/d\alpha$ respectively.
Taper and twist of each boomerang arm can be taken into account by choosing different values for α_o and chordlength at the tip and the root of the arm. Values of these quantities at other points of the arms are provided by linear interpolation. Since the arms may have parts of their leading and trailing edges interchanged during a part of a period of rotation, the characteristics of the reversed profiles should also be given. Hence we have for each boomerang arm the following set of parameters:

1. l = length of arm
2. e = excentricity of arm

3a. c_t = chordlength at tip

3b. c_r = chordlength at root

4a. α_{ot} = geometrical angle of incidence relative to zero lift at tip

4b. α_{or} = geometrical angle of incidence relative to zero lift at root

5. dC_L = slope of lift coefficient for $\alpha_1 < \alpha^* < \alpha_2$
6. C_{L1} = minimum lift coefficient ($\alpha^*=\alpha_1$)
7. C_{L2} = maximum lift coefficient ($\alpha^*=\alpha_2$)
8. C_{L3} = lift coefficient under which profile drag increases ($\alpha^*=\alpha_3$)
9. C_{L4} = lift coefficient above which profile drag increases ($\alpha^*=\alpha_4$)
10. C_{Dm} = minimum profile drag coefficient ($\alpha_3 \leq \alpha^* \leq \alpha_4$)
11. dC_D = slope of profile drag coefficient for $\alpha^* > \alpha_4$ or $\alpha^* < \alpha_3$

And additionally the quantities 4 through 11 for the reversed profile.

A somewhat more general model would allow the choice of different values at the tip and at the root also for the quantities 5 through 11. Our example boomerang (nr. 195) is characterized by the numbers listed in table 22.1. Lengths are given in millimeters, angles in degrees. In §24 graphs are shown of some lift distributions for this model boomerang.

arm	e	l	c_t	c_r	α_{ot}	α_{or}	dC_L	C_{L1}	C_{L2}	C_{L3}	C_{L4}	C_{Dm}	dC_D
1	+71	286	43	57	+5.0	+8.0	.10	-1.0	+2.0	-0.5	+1.0	.06	.03
	reversed profile				-1.0	+5.0	.10	-1.0	+2.4	-0.5	+0.8	.08	.03
2	-61	296	41	57	+2.0	+8.0	.10	-1.0	+2.0	-0.5	+1.0	.06	.03
	reversed profile				+2.0	+5.0	.10	-1.0	+2.4	-0.5	+0.8	.08	.03

table 22.1 Parameters characterizing model boomerang nr. 195 (lengths in mm., angles in degrees).

And additionally the quantities 4 through 11 for the reversed profile.

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	reversed profile				+2.0	+5.0	.10	-1.0	+2.4	-0.5	+0.8	.08	.03

table 22.1 Parameters characterizing model boomerang nr. 195 (lengths in mm., angles in degrees).

§23 *The use of ωa instead of V as reference velocity.*

Up to this point forces and torques have been made dimensionless by deviding them by $\mu V^2 a^2$ and $\mu V^2 a^3$ respectively (see §7). This is sensible with regard to fluid dynamics, where the velocity V of the undisturbed flow is used as a reference velocity (see (5.3)). With regard to the conditions of the flights of boomerangs, however, it may have certain advantages to use the boomerang's spin ω, or rather ωa, as a reference velocity. The forward velocity V of a boomerang may have large fluctuations during one flight, whereas a boomerang's spin ω remains relatively constant [Hess, 1968]. Moreover, during hovering, the forward velocity may even almost vanish. This means that $\Omega = \omega a/V$ may become very large.

If the resulting forces F_x, F_y, F_z and torques T_x, T_y, T_z are devided by $\mu\omega^2 a^4$ and $\mu\omega^2 a^5$ respectively, we obtain the dimensionless components F_{1x}, F_{1y}, F_{1z} (instead of F_{ox}, F_{oy}, F_{oz}) and T_{1x}, T_{1y}, T_{1z} (instead of T_{ox}, T_{oy}, T_{oz}). Thus:

$$F_{1x} = \frac{1}{\Omega^2} F_{ox}, \text{ etc.} \tag{23.1}$$

The behaviour of these new dimensionless components during a boomerang's flight resembles that of the real force components. For instance, if the forward speed V decreases, the forces generally decrease, where the previous dimensionless components generally would increase (Ω greater). A useful parameter, instead of the reduced spin Ω, is now:

$$U = \frac{1}{\Omega} = \frac{V}{\omega a} \tag{23.2}$$

which we call the advance ratio. This change of reference velocity does not in any way modify the methods developed in chapters I, II, III. The iteration procedure indicated in (21.7) is modified only to the extent that the criterion of convergence is now based on the computed new dimensionless components F_{1x}, F_{1y}, F_{1z}, T_{1x}, T_{1y}, T_{1z}. This criterion is relatively sharper for $U > 1$ ($\Omega < 1$) and wider for $U < 1$ ($\Omega > 1$). The iteration procedure (21.7) is finished if:

$$|F_{1x} \text{ new} - F_{1x} \text{ previous}| < CC, \text{ etc.} \tag{23.3}$$

where CC is a fixed number, used for all six components. A reasonable choice is $CC = 10^{-4}$.

The computed six dimensionless components are considered as functions of ψ and U. The advance ratio for real boomerangs may sometimes become very small; under such circumstances our semi-linear winglet model is not valid, of course, as v_z/V is not small then. However, on basis of symmetry we know that

$$\lim_{U \to o} F_{1x}, F_{1y}, T_{1x}, T_{1y} = 0 \tag{23.4}$$

and

$$\lim_{U \to o} F_{1z}, T_{1z} \text{ are independent of } \psi. \tag{23.5}$$

It would be difficult to express this useful information in terms of the old dimensionless components. In addition we have:

$$\lim_{\psi \to \pm \frac{1}{2}\pi} F_{1x}, F_{1y}, T_{1x}, T_{1y} = 0 \tag{23.6}$$

Finally, we describe in this section a modification specially related to boomerangs in free flight. A boomerang during its flight may not only rotate around the z-axis, but, due to precession, around the x- and y-axes as well. The influence of the latter angular velocities, ω_x and ω_y, on the lift distribution may be taken into account as follows. According to Part III, (5.13) we have for a boomerang in free flight:

$$\left.\begin{aligned} \frac{\omega_x}{\omega} &= -k \cdot T_{1y} \\ \frac{\omega_y}{\omega} &= +k \cdot T_{1x} \\ k &= \frac{\mu a^5}{I} \end{aligned}\right\} \tag{23.7}$$

Here I is the boomerang's moment of inertia about the z-axis, and k a dimensionless constant. (For ordinary boomerangs in air k is of the order of ½.) At each point in (ψ,U)-space ω_x and ω_y are given by (23.7), and $\vec{F}_1$ and $\vec{T}_1$ are still completely determined by ψ and U only:

$$\left.\begin{aligned}\vec{F} &= \mu\omega^2 a^4 \vec{F}_1^*(\psi,U)\\ \vec{T} &= \mu\omega^2 a^5 \vec{T}_1^*(\psi,U)\end{aligned}\right\}\quad (23.8)$$

The asterisks indicate that (23.7) is taken into account. This can be done by substituting in (2.15) and (9.6) for the winglet's velocity in a-direction:

$$W_z(x,y) = y\omega_x - x\omega_y \quad (23.9)$$

which can be written as:

$$\left.\begin{aligned}W_z(x,y) &= -k\omega(xT_{1x} + yT_{1y})\\ \frac{W_z(x,y)}{V} &= -\frac{k}{\Omega}\left(\frac{x}{a}T_{ox} + \frac{y}{a}T_{oy}\right)\end{aligned}\right\}\quad (23.10)$$

As W_z depends on the unknwon T_{1x} and T_{1y}, the solution for the forces and torques has to be found by iteration. It is easy to incorporate the present modification into the iteration scheme (21.7).

§24 *Lift function zero at leading edge of S.*

In §16 the suggestion was made that the $\frac{1}{2}(\pi-\varphi)$ term in the chordwise expansion of the load function $f_z(x,y)$ (see §5) might be omitted with common two-armed boomerangs. Frequently the lift distributions computed with this term gave the impression of too much "freedom" for the coefficients a_{0l} ($l = 1...N$), which would assume rather high values. These would be compensated for by the a_{pl} with $p = 1...M-1$, so that the resulting lower moments of the lift distribution came out reasonably, but the lift distribution would sometimes unduly fluctuate.

This section offers a comparison between computed lift distributions *with* and *without* the $\frac{1}{2}(\pi-\varphi)$ term. The calculations are based on the model boomerang nr. 195, characterized by table 22.1. The chordwise expansion (5.6) was used, but either with $p = 0...M-1$ or with $p = 1...M$. To compare the smoothness of the corresponding lift distributions we use the quantity $\overline{a}$ as defined by (15.3) if $p = 0...M-1$, and defined by:

$$\overline{a} \overset{\text{def}}{=\!=\!=} \frac{1}{MN} \sum_{p=1}^{M} \sum_{l=1}^{N} pl\,|a_{pl}| \qquad (24.1)$$

if $p = 1...M$. The computations were done with pivotal points according to (13.6), (13.7) and with $N = 8$, $M = 8$, $TOL = .01$, $CC = 10^{-4}$. Table 24.1 presents results for three selected cases: $\psi = 0°$ $U = 1$, $\psi = 0°$ $U = \frac{1}{2}$ and $\psi = 30°$ $U = \frac{1}{2}$ ($U = 1/\Omega$). The number 195 refers to the results computed with $p = 1...M$, and 195* to the computations with $p = 0...M-1$. Fig. 24.1 shows graphs of the lift distributions for these cases, produced in the same way as the graphs in §16.

The most striking difference between 195* and 195, as far as can be seen from table 24.1, concerns the magnitude of $\overline{a}$. Omitting the $\frac{1}{2}(\pi-\varphi)$ term reduces $\overline{a}$ by a factor of about 6! Even a superficial glance at the graphs in fig. 24.1 is sufficient to see that the $\frac{1}{2}(\pi-\varphi)$ terms strongly fluctuate in "spanwise" (y) direction. But, except for the region near the leading edge of the circular disk S, the lift distributions for 195* and 195 look very much the same. Although even for 195 the undulations in the graphs are probably partly artificial, it seems clearly

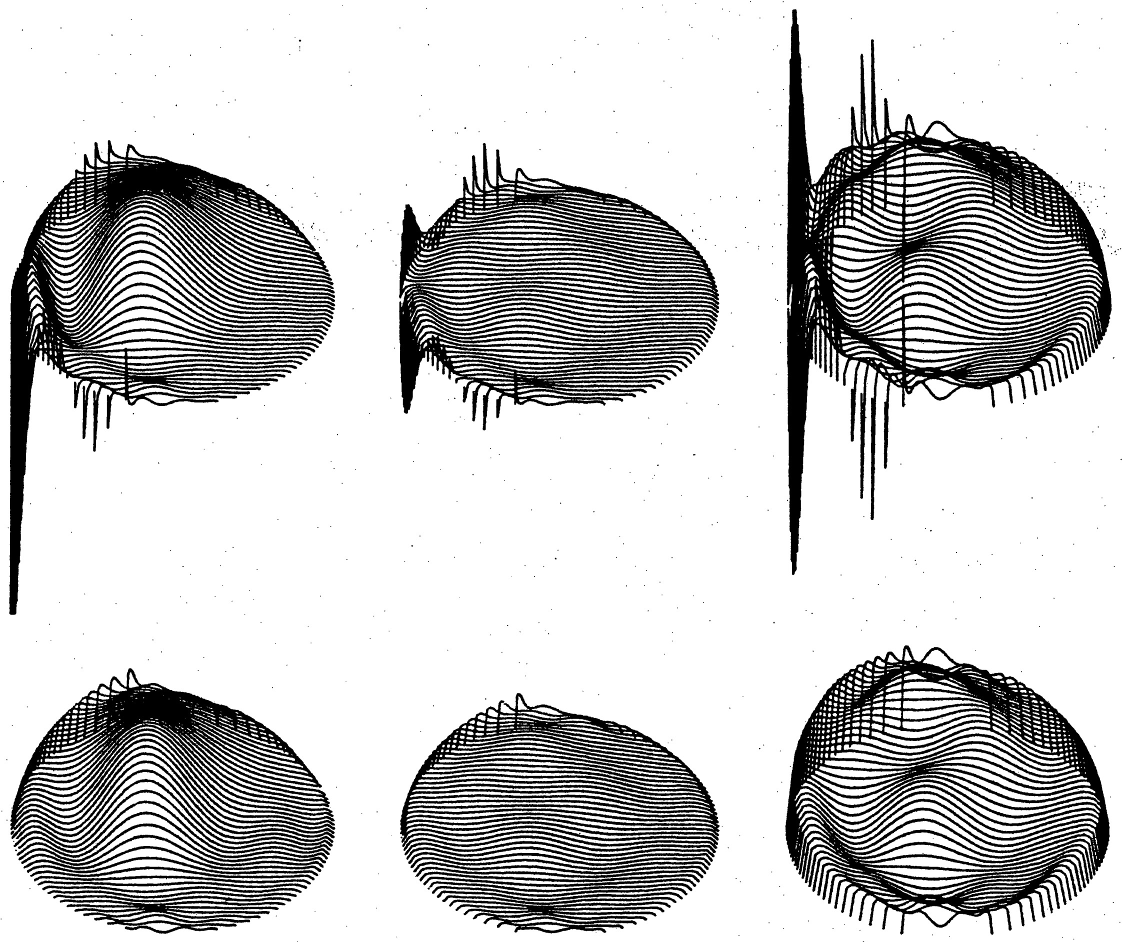

fig. 24.1. Top row: lift distributions computed with $p = 0 \ldots M-1$ (model boomerang 195*); bottom row: corresponding distributions computed with $p = 1 \ldots M$ (model boomerang 195). From left to right: $\psi = 0°$ $U = 1$, $\psi = 0°$ $U = \frac{1}{2}$, $\psi = 30°$ $U = \frac{1}{2}$ $(U = 1/\Omega)$. The computations were done with $N = 8$, $M = 8$, TOL = .01, CC = 10^{-4}. See table 22.1 for parameters characterizing model boomerang. Undisturbed flow from left to right, boomerang's rotation counterclockwise.
Verical scale at right corresponds to load function $f_z(x,y) = \frac{1}{4}(U = 1)$ resp. 1 $(U = \frac{1}{2})$.

	$\psi=0^\circ$ U=1		$\psi=0^\circ$ U=$\frac{1}{2}$		$\psi=30^\circ$ U=$\frac{1}{2}$	
	195*	195	195*	195	195*	195
F_{1z}	+ .0557	+ .0565	+ .0256	+ .0256	+ .1159	+ .1158
T_{1x}	+ .0175	+ .0175	+ .0073	+ .0072	+ .0259	+ .0257
T_{1y}	+ .0057	+ .0071	+ .0032	+ .0032	+ .0051	+ .0051
F_{1x}	+ .0075	+ .0076	+ .0029	+ .0029	– .0025	– .0024
F_{1y}	+ .0002	– .0001	+ .0001	+ .0000	– .0010	– .0010
T_{1z}	– .0043	– .0041	– .0028	– .0027	+ .0036	+ .0035
$\bar{a}$	.1113	.0154	.1576	.0208	.4856	.0729

table 24.1 Comparison between computations with p = 0...M-1 (195*) and with p = 1...M (195).

advisable on basis of these examples to omit the $\frac{1}{2}(\pi-\varphi)$ term as regards ordinary boomerangs. Table 24.1 indicates that the resulting force and torque components for 195* and 195 are not very much different. The most significant difference concerns the component T_{1y} (pitching moment); for $U \gtrsim 1$ it is greater for 195 (p = 1...M) than for 195* (p = 0...M-1). This is not only true for $\psi = 0^\circ$, but also for other values of ψ. The unphysically negative peak at the leading edge of S in the case 195*, $\psi = 0^\circ$ $U = 1$ (see fig. 24.1) yields a negative contribution to T_{1y}. This strongly suggests that the computed value for T_{1y} must be too low in this case.

From now on in this work the computed aerodynamic forces will be based on calculations with: p = 1...M, N = 8, M = 8, TOL = .01, CC = 10^{-4}.

§25 *The use of doubly cubic splines for interpolation.*

It is not feasible to compute the force and torque components acting on a theoretical boomerang at more than a restricted number of points in (ψ,U)-space. For example, the computations for the experiment described in §17 were made for the points determined by (17.2) and (17.3). Values of the components at other points of (ψ,U)-space can be obtained by interpolation. A suitable, smooth, two-dimensional interpolation is provided by doubly cubic splines. This section presents a description of the spline method, following Ahlberg, Nilson & Walsh [1967, chapters 2 and 7].

Two-dimensional doubly cubic interpolating splines can be derived from one-dimensional cubic interpolating splines in a simple manner. Therefore let us first consider these one-dimensional splines.

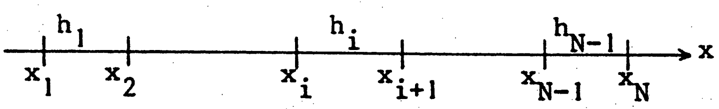

fig. 25.1 One-dimensional mesh.

Let the interval $[x_1,x_N]$ be devided into N-1 subintervals by the points $x_2 \dots x_{N-1}$ with $x_{i+1}-x_i = h_i > 0$, $i = 1 \dots N-1$. Thus the x-axis is devided into N+1 intervals which we number from 0 through N. Let f_i denote $f(x_i)$, $i = 1 \dots N$. f is the function which is to be interpolated, and its values f_i are given. The interpolating spline S is to be at most a cubic function of x on each of the subintervals i, $i = 1 \dots N-1$. We denote these functions by $S_i(x)$, $x_i \leq x \leq x_{i+1}$. The simplest choice for the end conditions is taking

$$M_1 = M_2, \; M_N = M_{N-1} \tag{25.1}$$

where

$$M_i = S''(x_i) \qquad i = 1 \dots N \tag{25.2}$$

Because of (25.1) $S_1(x)$ and $S_{N-1}(x)$ are quadratic functions. For $x < x_1$ and for $x > x_N$ we extrapolate f(x) by extrapolating $S_1(x)$ and $S_{N-1}(x)$ respectively.

It is convenient to work in terms of the moments M_i. Since the S_i are cubic functions:

$$S_i''(x) = M_i \frac{x_{i+1}-x}{h_i} + M_{i+1} \frac{x-x_i}{h_i} \qquad i = 1\ldots N-1 \qquad (25.3)$$

After integration and adjustment of the constants of integration (25.3) yields:

$$S_i(x) = \frac{x-x_i}{h_i} f_{i+1} + \frac{x_{i+1}-x}{h_i} f_i -$$

$$- \frac{h_i^2}{6} \cdot \frac{x-x_i}{h_i} \cdot \frac{x_{i+1}-x}{h_i} \left[\left(1 + \frac{x_{i+1}-x}{h_i}\right) M_i + \left(1 + \frac{x-x_i}{h_i}\right) M_{i+1}\right] \qquad (25.4)$$

and

$$S_i'(x) = \frac{f_{i+1}-f_i}{h_i} - \frac{M_{i+1}-M_i}{6} h_i - \tfrac{1}{2}M_i \frac{(x_{i+1}-x)^2}{h_i} + \tfrac{1}{2}M_{i+1} \frac{(x-x_i)^2}{h_i} \qquad (25.5)$$

S, S' and S" must be continuous for all x. This means

$$\left.\begin{aligned} S_i(x_i) &= S_{i-1}(x_i) \\ S_i'(x_i) &= S_{i-1}'(x_i) \\ S_i''(x_i) &= S_{i-1}''(x_i) \end{aligned}\right\} \quad i = 2\ldots N-1 \qquad (25.6)$$

The first and third of these equalities are automatically satisfied by (25.4) and (25.3). As to the second equality, we have

$$\left.\begin{aligned} S_i'(x_i) &= \frac{f_{i+1}-f_i}{h_i} - \frac{M_{i+1}-M_i}{6} h_i - \frac{M_i}{2} h_i \\ S_{i-1}'(x_i) &= \frac{f_i-f_{i-1}}{h_{i-1}} - \frac{M_i-M_{i-1}}{6} h_{i-1} + \frac{M_i}{2} h_{i-1} \end{aligned}\right\} \qquad (25.7)$$

Hence we must have

$$h_{i-1}M_{i-1} + 2(h_{i-1}+h_i)M_i + h_iM_{i+1} = 6\left(\frac{f_{i+1}-f_i}{h_i} - \frac{f_i-f_{i-1}}{h_{i-1}}\right), \quad i = 1\ldots N \qquad (25.8)$$

The equations (25.8) with the end conditions (25.1) can be solved by the following algorithm:

$$\left.\begin{array}{ll}
q_1 = 1,\ u_1 = 0, & \\
\left.\begin{array}{l}
z_i = 6\left(\dfrac{f_{i+1}-f_i}{h_i} - \dfrac{f_i-f_{i-1}}{h_{i-1}}\right) \\
p_i = h_{i-1}q_{i-1} + 2(h_{i-1}+h_i) \\
q_i = h_i/p_i \\
u_i = (z_i-h_{i-1}u_{i-1})/p_i
\end{array}\right\} & \text{for } i = 2\ldots N-1 \\
M_N = u_{N-1}/(1-q_{N-1}) & \\
M_i = M_{i+1}\cdot q_i+u_i & \text{for } i = N-1\ldots 1
\end{array}\right\} \quad (25.9)$$

By means of (25.9) the M_i can be computed from the f_i, this determines the spline S, which is given by (25.4).

It is convenient to construct a spline by the use of the cardinal splines $S^k(x)$, $k = 1\ldots N$, which are defined by

$$S^k(x_i) = \delta(i,k) \qquad i = 1\ldots N,\ k = 1\ldots N \qquad (25.10)$$

The interpolating spline S is then given by:

$$S = \sum_{k=1}^{N} S^k f_k \qquad (25.11)$$

The cardinal splines S^k only depend on the x_i, $i = 1\ldots N$. Their moments M_i^k can be determined by the algorithm (25.9) in which z_i is replaced by z_i^k:

$$z_i^k = \frac{6}{h_i}\,\delta(i+1,k) - \left(\frac{6}{h_i} + \frac{6}{h_{i-1}}\right)\delta(i,k) + \frac{6}{h_{i-1}}\,\delta(i-1,k) \qquad (25.12)$$

Let us now consider the two-dimensional case. A rectangular region of the (x,y)-plane is partitioned into rectangles by a mesh with meshpoints (x_i,y_j), $i = 1\ldots N$, $j = 1\ldots \overline{N}$, with $x_{i+1}-x_i = h_i > 0$ and $y_{j+1}-y_j = \overline{h}_j > 0$. We must determine the doubly cubic spline S which interpolates a function f(x,y) of which the values at the meshpoints (x_i,y_i) are given: $f_{i,j}$.

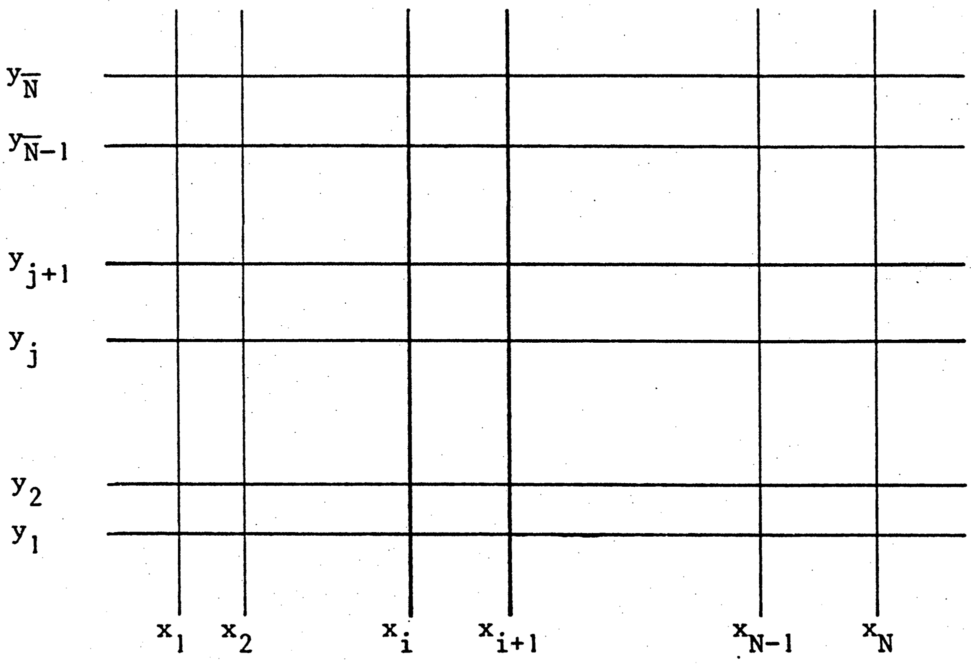

fig. 25.2 Two-dimensional mesh.

This spline can be expressed as:

$$S(x,y) = \sum_{k=1}^{N} \sum_{l=1}^{\bar{N}} f_{k,l} \; S^k(x)\bar{S}^l(y) \qquad (25.13)$$

where $S^k(x)$ and $\bar{S}^l(y)$ are the one-dimensional cardinal splines in x- and y-direction respectively. The one-dimensional cardinal splines in x-direction on the interval $x_i \leq x \leq x_{i+1}$, $i = 1\ldots N-1$ are given by:

$$S_i^k(x) = A_i\delta(i+1.k) + B_i\delta(i,k) + C_iM_{i+1}^k + D_iM_i^k, \quad k = 1\ldots N \qquad (25.14)$$

with

$$\left.\begin{aligned} A_i &= \frac{x-x_i}{h_i} \\ B_i &= \frac{x_{i+1}-x}{h_i} \\ C_i &= -\frac{h_i^2}{6}A_iB_i(1+A_i) \\ D_i &= -\frac{h_i^2}{6}A_iB_i(1+B_i) \end{aligned}\right\} \qquad (25.15)$$

The moments M_i^k, $i = 1 \ldots N$, $k = 1 \ldots N$ follow from (25.9). In a completely analogous way we have for the one-dimensional cardinal splines in y-direction on the interval $y_j \leq y \leq y_{j+1}$, $j = 1 \ldots \overline{N}-1$:

$$\overline{S}_j^l(y) = \overline{A}_j \delta(j+1,l) + \overline{B}_j \delta(j,l) + \overline{C}_j \overline{M}_{j+1}^l + \overline{D}_j \overline{M}_j^l . \qquad l = 1 \ldots \overline{N} \tag{25.16}$$

with

$$\left.\begin{aligned}
\overline{A}_j &= \frac{y-y_j}{\overline{h}_j} \\
\overline{B}_j &= \frac{y_{j+1}-y}{\overline{h}_j} \\
\overline{C}_j &= -\frac{\overline{h}_j^2}{6} \overline{A}_j \overline{B}_j (1+\overline{A}_j) \\
\overline{D}_j &= -\frac{\overline{h}_j^2}{6} \overline{A}_j \overline{B}_j (1+\overline{B}_j)
\end{aligned}\right\} \tag{25.17}$$

Again the moments $\overline{M}_j^l$, $j = 1 \ldots \overline{N}$, $l = 1 \ldots \overline{N}$ follow from (25.9). The two-dimensional doubly cubic interpolating spline S on the rectangle $x_i \leq x \leq x_{i+1}$, $y_j \leq y \leq y_{j+1}$ is given by

$$S_{i,j}(x,y) = \sum_{k=1}^{N} \sum_{l=1}^{\overline{N}} f(x_k,y_l) S_i^k(x) \overline{S}_j^l(y) \qquad i = 1 \ldots N-1, \; j = 1 \ldots \overline{N}-1 \tag{25.18}$$

Substitution of (25.14) through (25.17) into (25.18) yields:

$$\begin{aligned}
S_{i,j}(x,y) = {} & A_i \overline{A}_j f(i+1,j+1) + A_i \overline{B}_j f(i+1.j) + B_i \overline{A}_j f(i,j+1) + B_i \overline{B}_j f(i,j) + \\
& + A_i \overline{C}_j P(i+1,j+1) + A_i \overline{D}_j P(i+1.j) + B_i \overline{C}_j P(i,j+1) + B_i \overline{D}_j P(i,j) + \\
& + C_i \overline{A}_j Q(i+1,j+1) + C_i \overline{B}_j Q(i+1,j) + D_i \overline{A}_j Q(i,j+1) + D_i \overline{B}_j Q(i,j) + \\
& + C_i \overline{C}_j R(i+1,j+1) + C_i \overline{D}_j R(i+1,j) + D_i \overline{C}_j R(i,j+1) + D_i \overline{D}_j R(i,j)
\end{aligned} \tag{25.19}$$

with: